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BIOMETRIKA.

ON THE SYSTEMATIC FITTING OF CURVES TO OBSERVATIONS AND MEASUREMENTS.

PART II.*

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(9) *Illustration V.* In the previous illustration we have fitted the best curve not to the ordinates, but to the logarithms of the ordinates. This method was forced upon us by the complexity of Makeham's formula. It will clearly give good results in many cases where it might be difficult to calculate the moments of the ordinates of the curve, but in which the moments of the logarithms of the ordinates follow quite easily. For example in curves of the type $y = e^{f(x)}$ we shall often introduce considerable simplicity into our work without loss of practical efficiency by fitting to the data a curve $Y = f(x)$, where $Y = \log y$. In particular

* Part I. was published in Vol. I. p. 265.

if $f(x)$ be an algebraical expression in integer powers of x, we reduce the fitting of curves of this type to the theory of parabola-fitting with which we shall be occupied later.

A very interesting case of such work arises in dealing with frequency distributions, which we suppose to be normal or approximately normal, but of which only a portion of the distribution can be known or observed. For example, the marks of candidates in a competitive examination, wherein candidates below a certain grade have been rejected by a preliminary examination, or are cast out without placing. Or again, the statures of the soldiers in a regiment with a minimum admissible height.

Clearly in such cases as these we have to fit a curve

$$y = y_0 e^{-(ax^2+bx)},$$

given a certain number only of values of y and x.

The method of least squares or that of moments would enable us theoretically to determine y_0, a and b and so to find the constants of the best fitting normal distribution. But with the curve in its above form the equations, especially in the case of least squares, become unmanageable. If, however, we write $y = e^Y$, we find the problem reduces to fitting

$$Y = a'x^2 + b'x + c',$$

where Y is known for a certain range of values of x.

As far as I know the first attempt to determine the constants of a normal curve when only a portion of the distribution is known was made by Mr Francis Galton in his memoir on the speed distribution of American Trotting Horses*. The American record contains only horses which can trot a mile in less than a given number of seconds. Hence assuming the distribution to be normal we obtain only a portion of the frequency distribution, i.e. the number of horses that can trot a mile in each number of seconds less than this maximum.

Taking a normal curve

$$y = y_0 e^{-\frac{(x-h)^2}{2\sigma^2}},$$

Mr Galton has determined the position of the mode, i.e. the value of h, only by inspection of the plotted figures. It seemed worth while to compare his results with what we should get by fitting curves

$$Y = a'x^2 + b'x + c'$$

to the logarithms of his frequency data, using the method of fitting parabolas of the second order discussed on p. 14 below†.

It seems well to briefly indicate the process used. The curve for the year 1893 was determined by me, those for 1892, 1894 and 1895 I owe entirely to the energy of Mr Leslie Bramley-Moore.

* *R. S. Proc.* Vol. 62, p. 310.

† The curve being parabolic the methods of moments and of least squares are now sensibly identical in result, although not alike in their stages.

Taking Mr Galton's polygon for the data of 1892 nineteen ordinates were obtained for speeds at equal intervals of a second, 29—28, 28—27 ... 11—10 from the observations on 1324 trotters. On an arbitrary scale these nineteen ordinates are given in Column (1). In Column (2) are their logarithms to three figures. Column (3) gives the first moment m_1 about the middle of the range $2l = 19$. Column (4) the second moment m_2 about the same point. From m_0, m_1, m_2 and l, λ_0, λ_1 and λ_2 were found and hence e_0, e_1 and e_2 (see p. 14).

(1) y	(2) Y	x	(3) $S(xY)$	(4) $S(x^2Y)$
92·8	1·968	−9	−17·712	159·408
100·4	2·002	−8	−16·016	128·128
95·0	1·978	−7	−13·846	96·922
71·2	1·852	−6	−11·112	66·672
67·6	1·830	−5	− 9·150	45·750
61·3	1·787	−4	− 7·148	28·592
61·4	1·788	−3	− 5·364	16·092
44·8	1·651	−2	− 3·302	6·604
44·5	1·648	−1	− 1·648	1·648
45·8	1·661	0	−85·298	
38·4	1·584	+1	1·584	1·584
27·8	1·444	+2	2·888	5·776
19·8	1·297	+3	3·891	11·673
10·7	1·029	+4	4·116	16·464
15·8	1·199	+5	5·995	29·975
7·9	·898	+6	5·388	32·328
5·0	·699	+7	4·893	34·251
2·1	·322	+8	2·576	20·608
5·6	·748	+9	6·732	60·588
	$m_0 = 27·385$		+38·063	$m_2 = 763·063$
			−85·298	
			$m_1 = -47·235$	

Thus $$y_0 = \frac{m_0}{2l} = 1·441,316, \quad \lambda_1 = \frac{m_1}{m_0 l} = -·181,564,$$

$$\lambda_2 = \frac{m_2}{m_0 l^2} = ·308,7454.$$

Whence $e_0 = 1·092,205$, $e_1 = -·544,692$, $e_2 = -·276,6143$.

These give us for the required parabola:

$$Y = 1·441,316 \left\{1·092,205 - ·544,692 \left(\frac{x}{l}\right) - ·276,614 \left(\frac{x}{l}\right)^2\right\}.$$

This may be thrown into the form:

$$Y = 1·441,316 \left\{1·360,583 - ·276,614 \left(\frac{x + \bar{x}}{l}\right)^2\right\},$$

where $$\bar{x} = -9·3575.$$

Now l is 9·5; hence the centre of the normal curve is at ·1425 beyond the start of the range which is 29 secs., i.e. 28·8575 secs. is the modal speed of the group of trotters.

If the normal curve be

$$y = z_0 e^{-\frac{(x+\bar{x})^2}{2\sigma^2}},$$

$$Y = \log y = \log z_0 - \frac{(x+\bar{x})^2}{2\sigma^2} \log e.$$

Whence we find

$$\log z_0 = 1{\cdot}441{,}316 \times 1{\cdot}360{,}583,$$

$$2\sigma^2 = \frac{l^2 \log e}{1{\cdot}441{,}316 \times {\cdot}276{,}614},$$

and ultimately

$$z_0 = 9{\cdot}14176, \quad \sigma = 7{\cdot}011{,}073.$$

Thus the required normal curve is

$$y = 9{\cdot}14176\, e^{-\frac{1}{2}x^2/(7{\cdot}011{,}073)^2}.$$

The "probable error" corresponding to the above value of σ is 4·7289, which enables us to compare our results directly with Mr Galton's numbers.

Like Mr Galton we have omitted from consideration the group of horses with speeds between 29 and 30 secs., for it includes a large number of doubtful trotters, whose speed is allowed by grace to fall within the 30-second limit.

The following are the actual determinations for four years of the 'centre' and variability of American trotters. They are compared with Mr Galton's determinations by inspection of the 'centre' and his calculation from this of the probable error by quartiles.

Year	Constant	Moments	Galton
1892	Centre	28·8575	29
	S. D.	7·0111	—
	P. E.	4·7289	4·25
1893	Centre	28·0120	26
	S. D.	8·8345	—
	P. E.	5·9588	5·0
1894	Centre	25·6256	27
	S. D.	6·2946	—
	P. E.	4·2456	4·5
1895	Centre	28·0119	27
	S. D.	8 5459	—
	P. E.	5·7641	5·0

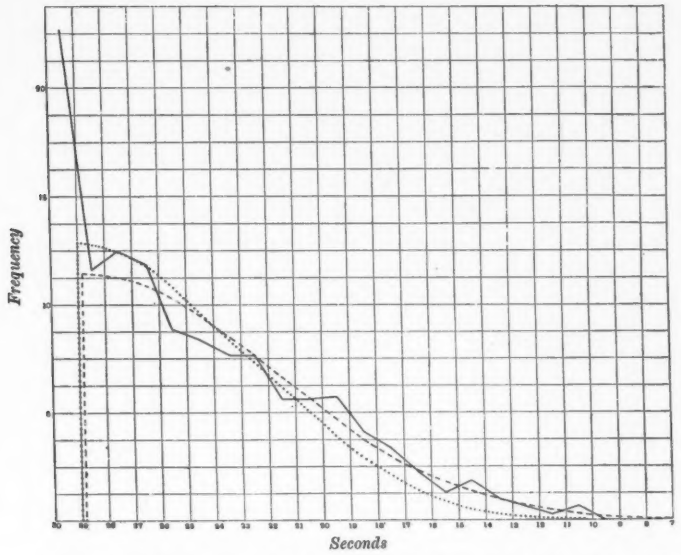


FIG. 5. Distribution of Speed of American Trotters, 1892. 1324 Observations.

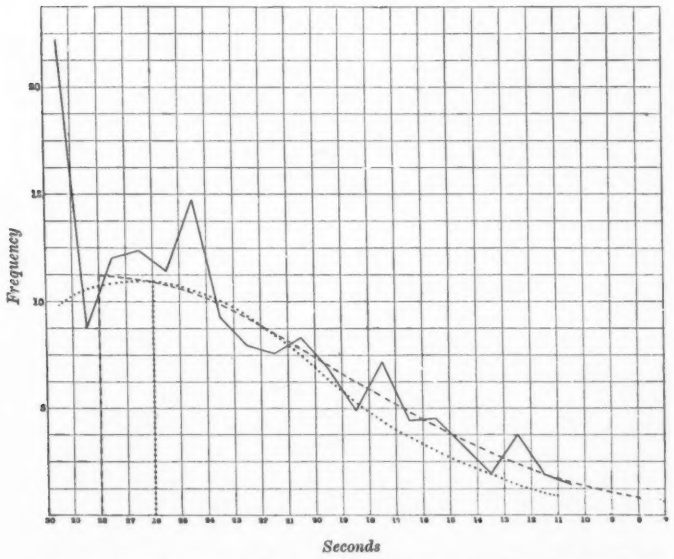


FIG. 6. Distribution of Speed of American Trotters, 1893. 982 Observations.
Moments - - - - Inspection and Quartile .....

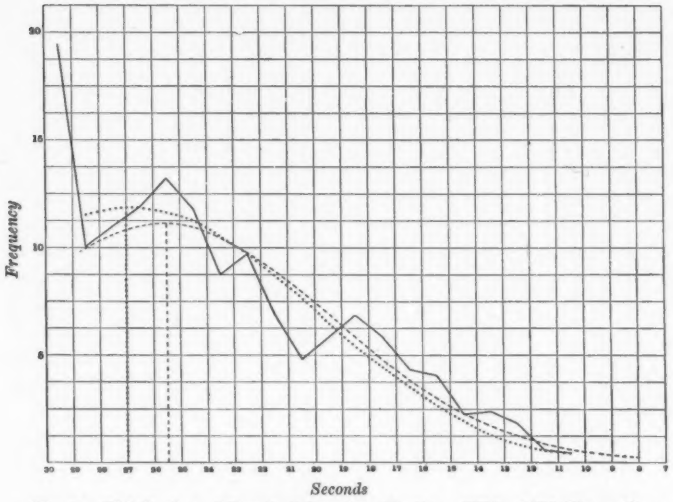


FIG. 7. Distribution of Speed of American Trotters, 1894. 1204 Observations.

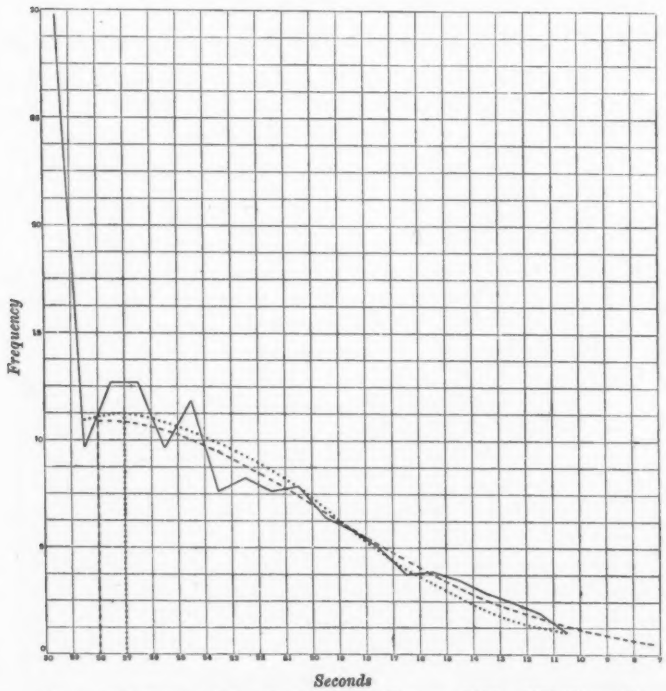
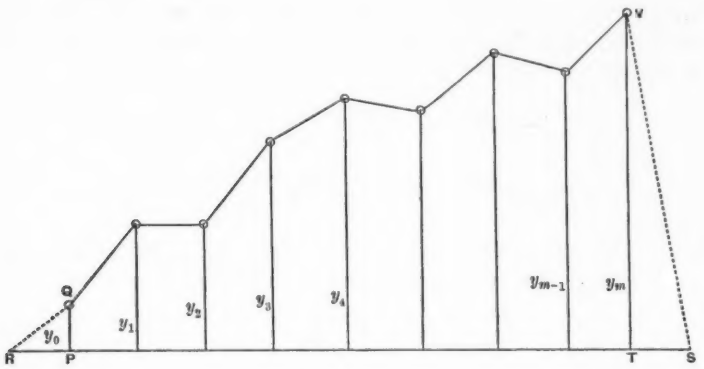


FIG. 8. Distribution of Speed of American Trotters, 1895. 1124 Observations.
Moments - - - - Inspection and Quartile .....

A glance at the accompanying Figs. 5, 6, 7 and 8 will show that the fit has been much improved by adopting the systematic method of moments. At the same time using a Brunsviga for all multiplications and divisions and a Comptometer for additions the labour is not very severe. Of course it is not contended that this accuracy is necessary in the present case; Mr Galton's approximations are probably close enough for the ends he had in view. We have only used his data to illustrate a method, which may be of service for special cases, where the best available determinations of the constants are needed.

(10) In calculating the moments in the previous illustration we have simply concentrated along the mid-ordinates. This was close enough for the purpose of illustration. When the ordinates of a curve which requires fitting are true ordinates, say for example, measurements obtained by observation, their irregularity is often such that it appears idle to use complex quadrature formulæ. Such formulæ give very good results, if the ordinates are those of a mathematical formula, or if we have a fairly smooth system of points. But a very frequent case is a case like that of the accompanying figure, in which a quad-



rature formula for the moments seems idle and yet we are scarcely justified in concentrating along the ordinates to find the moments, if the base unit be not indefinitely small. Here it seems reasonable to take the area and moments of the system of trapezia as fairly representing the area and moments of the curve to be fitted. It would be idle to use a formula like Simpson's, for example; because the changes in curvature in the curve, which would be allowed for by Simpson's method of striking a parabola through three successive points, have clearly no existence in the general sweep of the observations and are due only to irregularities of observation.

Accordingly we want an expression for the moments of a system of trapezia. Let the ordinates be $y_0, y_1, y_2 \ldots y_m$ and the corresponding abscissæ $x_0, x_1, x_2 \ldots x_m$.

We shall limit our discussion to equal base elements b, which can always be taken as unity.

Let $$\nu_n' = \sum_0^m (x^n y).$$

Then the following expression for the nth moment M_n' of the trapezoidal area about the axis of y is easily deduced:

$$M_n' = \nu_n' b + \frac{n(n-1)}{12}\nu'_{n-2} b^2 + \frac{n(n-1)(n-2)(n-3)}{360}\nu'_{n-4} b^4$$
$$+ \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{20160}\nu'_{n-6} b^6 + \dots$$
$$- y_m \left\{ \frac{x_m^{\,n}}{\lfloor 2} b + \frac{n}{\lfloor 3} x_m^{\,n-1} b^2 + \frac{n(n-1)}{\lfloor 4} x_m^{\,n-2} b^3 + \dots \right\}$$
$$- y_0 \left\{ \frac{x_0^{\,n}}{\lfloor 2} b - \frac{n}{\lfloor 3} x_0^{\,n-1} b^2 + \frac{n(n-1)}{\lfloor 4} x_0^{\,n-2} b^3 - \dots \right\}.$$

Here the "corrective terms" in y_m and y_0 are nothing more than the subtraction of the nth moments of the triangles PQR and STV from the nth moment of the whole figure on base RS, which is represented by the remainder of the expression. They can be thrown into the simple forms

$$- y_m \frac{(x_m + b)^{n+2} - (n+2)\, x_m^{\,n+1} b - x_m^{\,n+2}}{(n+1)(n+2)},$$

$$- (-1)^n y_0 \frac{(-x_0 + b)^{n+2} - (n+2)(-x_0)^{n+1} b - (-x_0)^{n+2}}{(n+1)(n+2)}.$$

Now let $PT = 2l$ and let us take moments about the middle of PT. Let M_n be the nth moment about this point, and x being measured from it, let

$$\nu_n = \sum_0^m (x^n y).$$

We have $$x_m = -x_0 = l,$$

and accordingly, if l be measured in b as unit,

$$M_n = \nu_n + \frac{n(n-1)}{12}\nu_{n-2} + \frac{n(n-1)(n-2)(n-3)}{360}\nu_{n-4}$$
$$+ \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{20160}\nu_{n-6} + \dots\dots$$
$$- \frac{(l+1)^{n+2} - l^{n+2} - (n+2)\, l^{n+1}}{(n+1)(n+2)} \{y_m + (-1)^n y_0\}.$$

The corrective term is thus very simple: it may be written

$$L_n (y_m + (-1)^n y_0),$$

where L_n may be calculated once for all for $n = 0$ to 5 and for values of $l = 3$ to 20 or for all odd numbers of ordinates from 7 to 41. These values are given in the accompanying Table, which will enable the reader to readily find the corrective term, bearing in mind that, if the moment is even, L_n must be multiplied by $y_m + y_0$, and that if it is odd it must be multiplied by $y_m - y_0$.

Table of Corrective Terms for Moments of Trapezia.

$$L_n = \frac{(l+1)^{n+2} - (n+2)l^{n+1} - l^{n+2}}{(n+1)(n+2)}.$$

l	L_0	L_1	L_2	L_3	L_4	L_5
3	·5	1·667	5·583	18·800	63·633	216·524
4	·5	2·167	9·417	41·050	179·500	787·357
5	·5	2·667	14·250	76·300	409·367	2200·857
6	·5	3·167	20·083	127·550	811·233	5167·024
7	·5	3·667	26·917	197·800	1455·100	10715·857
8	·5	4·167	34·750	290·050	2422·967	20257·357
9	·5	4·667	43·583	407·300	3808·833	35641·524
10	·5	5·167	53·417	552·550	5718·700	59218·357
11	·5	5·667	64·250	728·800	8270·567	93897·857
12	·5	6·167	76·083	939·050	11594·433	143210·024
13	·5	6·667	88·917	1186·300	15832·300	211364·857
14	·5	7·167	102·750	1473·550	21138·167	303312·357
15	·5	7·667	117·583	1803·800	27678·033	424802·524
16	·5	8·167	133·417	2180·050	35629·900	582445·357
17	·5	8·667	150·250	2605·300	45183·767	783770·857
18	·5	9·167	168·083	3082·550	56541·633	1037289·024
19	·5	9·667	186·917	3614·800	69917·500	1352549·857
20	·5	10·167	206·750	4205·050	85537·367	1740203·357

(11) *On the fitting of Parabolic Curves to given Data.*

Let us consider this problem first from the standpoint of the method of least squares.

Let the parabola be of the $(n-1)$th order and represented by

$$y = c_0 + c_1 x + c_2 x^2 + \ldots + c_{n-1} x^{n-1}.$$

Let us write for brevity

$$\nu_r' = S(x^r y),$$
$$\lambda_r' = S(x^r),$$

where S denotes a summation extending to the values of x and y for every observation. Then proceeding to make

$$S(y - c_0 - c_1 x - c_2 x^2 - \ldots - c_{n-1} x^{n-1})^2$$

a minimum for values of $c_0, c_1, c_2 \ldots c_{n-1}$ we easily find the type equations

$$\nu_0' = c_0\lambda_0' + c_1\lambda_1' + c_2\lambda_2' + \ldots + c_{n-1}\lambda'_{n-1}$$
$$\nu_1' = c_0\lambda_1' + c_1\lambda_2' + c_2\lambda_3' + \ldots + c_{n-1}\lambda'_n$$
$$\nu_2' = c_0\lambda_2' + c_1\lambda_3' + c_2\lambda_4' + \ldots + c_{n-1}\lambda'_{n+1}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\nu'_{n-1} = c_0\lambda'_{n-1} + c_1\lambda'_n + c_2\lambda'_{n+1} + \ldots + c_{n-1}\lambda'_{2n-1}.$$

These are n equations to find the n constants.

Now in any curve-fitting with n constants to be found we must determine the area and moments $\nu_0', \nu_1', \nu_2' \ldots \nu'_{n-1}$; but we find that the method of least squares involves also the discovery of the $2n$ quantities $\lambda_0', \lambda_1', \lambda_2' \ldots \lambda'_{2n-1}$.

If as is very usually the case the observations are taken at equal intervals and there are m of them, we can by choosing this interval as our unit and a proper origin, write

$$\lambda_r' = \Sigma(x^r) = 1^r + 2^r + 3^r + \ldots + m^r,$$

or λ_r' is the sum of the rth powers of the first m natural numbers. Thus the values of λ_r' for successive values of m can be tabled once and for all. This is done for $m = 1$ to 20 and $r = 1$ to 7 in the accompanying Table.

Table of the Sums of the first seven Powers of the Natural Numbers.*

Up to	$S(x)$	$S(x^2)$	$S(x^3)$	$S(x^4)$	$S(x^5)$	$S(x^6)$	$S(x^7)$
1	1	1	1	1	1	1	1
2	3	5	9	17	33	65	129
3	6	14	36	98	276	794	2316
4	10	30	100	354	1300	4890	18700
5	15	55	225	979	4425	20515	96825
6	21	91	441	2275	12201	67171	376761
7	28	140	784	4676	29008	184820	1200304
8	36	204	1296	8772	61776	446964	3297456
9	45	285	2025	15333	120825	978405	8080425
10	55	385	3025	25333	220825	1978405	18080425
11	66	506	4356	39974	381876	3749966	37567596
12	78	650	6084	60710	630708	6735950	73399404
13	91	819	8281	89271	1002001	11562759	136147921
14	105	1015	11025	127687	1539825	19092295	241561425
15	120	1240	14400	178312	2299200	30482920	412420800
16	136	1496	18496	243848	3347776	47260136	680856256
17	153	1785	23409	327369	4767633	71397705	1091194929
18	171	2109	29241	432345	665720[illegible]	105409929	1703414961
19	190	2470	36100	562666	9133300	152455810	2597286700
20	210	2870	44100	722666	12333300	216455810	3877286700

* Mr W. Palin Elderton has calculated an extended table of this kind, giving the sums of the 7th powers of all the natural numbers up to 100. I hope it may be possible eventually to reproduce and distribute this much enlarged table.

It will be seen at once that λ_7' is very large for any number of observations greater than 10. Even if we go as far as λ_7', we shall, however, be only fitting a parabola of the *third* order, and the four type equations to be solved will be found as a rule even in this case to be rather unmanageable. For parabolas of the fourth, fifth and sixth orders, the labour becomes very severe.

It is clear however that if we evaluated the determinant

$$\begin{vmatrix} \lambda_0', & \lambda_1', & \lambda_2', & \dots \lambda'_{n-1} \\ \lambda_1', & \lambda_2', & \lambda_3', & \dots \lambda'_n \\ \lambda_2', & \lambda_3', & \lambda_4', & \dots \lambda'_{n+1} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \lambda'_{n-1}, & \lambda'_n, & \lambda'_{n+1}, & \dots \lambda'_{2n-1} \end{vmatrix}$$

and its minors for values of $n = 1, 2, \dots 5, 6$, and of m from 1 up to 20, we should have a set of constants which would enable us readily to find $c_0, c_1, \dots$ as soon as the $\nu_0', \nu_1', \dots \nu'_{n-1}$ were given. The arithmetical work to be *once* done would be considerable, but it might be worth doing, supposing the method of moments were not available with a simpler solution.

It may be noted that a considerable simplification of the least square type equations can be introduced if there be *an odd number* of observations. Let us take the origin at the middle observation, then clearly

$$\lambda_1' = \lambda_3' = \lambda_5' = \dots = 0,$$

or all the odd x sums vanish. Let us use undashed letters to denote moments about the centre of the range, then we find our system of type equations breaks up into two

$$\begin{aligned} \nu_0 &= c_0\lambda_0 + c_2\lambda_2 + c_4\lambda_4 + \\ \nu_2 &= c_0\lambda_2 + c_2\lambda_4 + c_4\lambda_6 + \\ \nu_4 &= c_0\lambda_4 + c_2\lambda_6 + c_4\lambda_8 + \\ &\dots\dots\dots\dots \end{aligned} \qquad \text{and} \qquad \begin{aligned} \nu_1 &= c_1\lambda_2 + c_3\lambda_4 + c_5\lambda_6 + \\ \nu_3 &= c_1\lambda_4 + c_3\lambda_6 + c_5\lambda_8 + \\ \nu_5 &= c_1\lambda_6 + c_3\lambda_8 + c_5\lambda_{10} + \\ &\dots\dots\dots\dots \end{aligned}$$

Our Table will now enable us to find λ_6 for any number of observations up to 41; and for parabolas of the third order only, we have simply to solve two sets of linear equations, each of which contains only two unknowns. Thus the work becomes extremely simple. This is in fact how the cubical parabola was fitted by the method of least squares to the observations in our illustration in § 3 (Vol. I. p. 280).

On the other hand even with this choice of origin we require λ_8, λ_{10} and λ_{12} to fit parabolas of the fourth, fifth and sixth orders; and the sum of the 10th or 12th powers of the natural numbers* and simultaneous equations with three or four variables and coefficients of very diverse magnitudes are rather troublesome matters to deal with.

* Of course λ_r may be calculated from the Bernoulli number formula on p. 286, *Biometrika*, Vol. I.

Of course if the λ determinant and its minors were once worked out, say for the first six parabolas and for the customary range of values of m, we should have no more labour, but meanwhile it seems to me that the method of least squares must for practical purposes be laid on one side even for parabolic curves except in the simple cases of those of the first, second and third orders. But if the method of least squares be of small practical use even in this one of the simplest cases of curve-fitting, it may be questioned whether it is not better to adopt the uniform process of moments throughout.

(12) Let us now apply the method of moments to the parabolic curve

$$y = e_0 + e_1 x + e_2 x^2 + \ldots + e_{n-1} x^{n-1}$$

for which the expansion by Maclaurin's Theorem is exact. Let $2l$ be the range for which this curve is to be fitted to the observations, and let us take the origin at the mid-point of the range.

Let m_0 be the area and $m_1, m_2 \ldots m_{n-1}$ the first n moments of the observation polygon about the axis of y, i.e. the perpendicular to the range at its mid-point. Let us write $m_0 = 2l \times y_0$, so that y_0 is the mean value of the ordinate. Then the curve to be fitted may be written in the form

$$y = y_0 \left(e_0 + e_1 \frac{x}{l} + e_2 \left(\frac{x}{l}\right)^2 + \ldots + e_{n-1} \left(\frac{x}{l}\right)^{n-1} \right).$$

Multiply by $\left(\frac{x}{l}\right)^{2r}$ and integrate from $x = l$ to $x = -l$:

$$m_{2r}/l^{2r} = 2y_0 l \left\{ \frac{e_0}{2r+1} + \frac{e_2}{2r+3} + \ldots + \frac{1-(-1)^n}{2} \frac{e_{n-1}}{2r+n} \right\}.$$

If we multiply by $(x/l)^{2r+1}$ and integrate,

$$m_{2r+1}/l^{2r+1} = 2y_0 l \left\{ \frac{e_1}{2r+3} + \frac{e_3}{2r+5} + \ldots + \frac{1+(-1)^n}{2} \frac{e_{n-1}}{2r+n+1} \right\}.$$

It is obvious that the even e's will be given in terms of the even moments and the odd e's in terms of the odd moments by two independent series of equations.

Let us write $\lambda_s = m_s/(m_0 l^s)$, thus $\lambda_0 = 1$.

Then

$$\begin{aligned}
\lambda_0 &= e_0 + \tfrac{1}{3} e_2 + \tfrac{1}{5} e_4 + \ldots \\
\lambda_2 &= \tfrac{1}{3} e_0 + \tfrac{1}{5} e_2 + \tfrac{1}{7} e_4 + \ldots \\
\lambda_4 &= \tfrac{1}{5} e_0 + \tfrac{1}{7} e_2 + \tfrac{1}{9} e_4 + \ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\lambda_1 &= \tfrac{1}{3} e_1 + \tfrac{1}{5} e_3 + \tfrac{1}{7} e_5 + \ldots \\
\lambda_3 &= \tfrac{1}{5} e_1 + \tfrac{1}{7} e_3 + \tfrac{1}{9} e_5 + \ldots \\
\lambda_5 &= \tfrac{1}{7} e_1 + \tfrac{1}{9} e_3 + \tfrac{1}{11} e_5 + \ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

Hence it is clear that any e can be expressed at once in terms of the moments and of one or other of the determinants

$$\begin{vmatrix} 1, & 1/3, & 1/5, & 1/7, & \dots \\ 1/3, & 1/5, & 1/7, & 1/9, & \dots \\ 1/5, & 1/7, & 1/9, & 1/11, & \dots \\ \dots & \dots & \dots & \dots & \\ \dots & \dots & \dots & \dots & \end{vmatrix}, \quad \begin{vmatrix} 1/3, & 1/5, & 1/7, & 1/9, & \dots \\ 1/5, & 1/7, & 1/9, & 1/11, & \dots \\ 1/7, & 1/9, & 1/11, & 1/13, & \dots \\ \dots & \dots & \dots & \dots & \\ \dots & \dots & \dots & \dots & \end{vmatrix}$$

and their respective minors.

It is thus quite easy to express the general result of working with $e_0, e_1 \dots e_{n-1}$. But as a matter of practical application, it would involve far too much troublesome arithmetic to calculate moments beyond the fifth or sixth. We have accordingly only to calculate once and for all the numerical coefficients of the λ's in the values of the e's for the first few cases and these will serve for all future applications.

Case (i). *To fit a straight line to a series of observations.*

Let the line be $$y = y_0\left(e_0 + e_1\frac{x}{l}\right).$$

Then $$\lambda_0 = e_0, \quad \lambda_1 = \tfrac{1}{3}e_1.$$

Thus the equation of the line is

$$y = y_0\left(\lambda_0 + 3\lambda_1\frac{x}{l}\right).$$

Geometrical Construction. Let the broken line AB be the observations and $A'B'$ the best straight line. Then $A'B'EF$ and $ADBEF$ must have the same first moments and the same area. Let CK be the vertical through the centroid of the observations, i.e. obtained by taking their mean FK. Now the trapezium may be considered as made up of two triangles $A'EF$ and $A'B'E$, the centroids of which lie in the vertical lines G_1H_1 and G_2H_2 trisecting FE. Hence the area $A'B'EF$ acting in CK must be equivalent to the areas $A'EF$ and $A'B'E$, or $l \times A'F$ and $l \times B'E$, acting in G_1H_1 and G_2H_2. Now $A'F + B'E$ is known, for it equals $2y_0$, the double of the mean ordinate of the observations.

Take $01 = 2y_0$ and from any point O, draw $O0$ to meet G_1H_1 and CK in t and u. Draw uv parallel to $O1$ to meet G_2H_2 in v, and then draw $O2$ parallel to tv to meet 01 in 2. We shall then have $02 = A'F$ and $21 = B'E$, the required lengths, which fully determine the line $A'B'$.

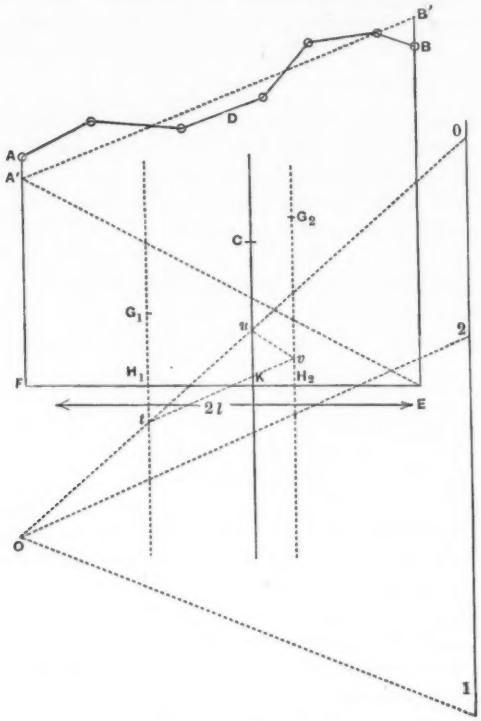
The construction given is the familiar graphical one for finding the components in the lines G_1H_1 and G_2H_2 parallel to CK of a force $2y_0$ acting in CK. The principle of moments would also give a solution. Thus take moments about H_2:

$$A'F \times \frac{2l}{3} = 2y_0 \times H_2K,$$

whence $$A'F = \frac{3y_0}{l} \times H_2K,$$

similarly $$B'E = \frac{3y_0}{l} \times H_1K,$$

which determine the intercepts since H_1K and H_2K are known.



Case (ii). *To fit a parabola of the second order to a series of observations.*

Let the parabola be $$y = y_0\left\{e_0 + e_1\frac{x}{l} + e_2\left(\frac{x}{l}\right)^2\right\}.$$

Our equations are now

$$\lambda_0 = e_0 + \tfrac{1}{3}e_2,$$
$$\lambda_2 = \tfrac{1}{3}e_0 + \tfrac{1}{5}e_2,$$
$$\lambda_1 = \tfrac{1}{3}e_1.$$

Hence

$$e_0 = \tfrac{3}{4}(3\lambda_0 - 5\lambda_2),$$
$$e_1 = 3\lambda_1,$$
$$e_2 = \tfrac{15}{4}(3\lambda_2 - \lambda_0).$$

Thus e_0, e_1, e_2 are at once found, when λ_0, λ_1 and λ_2 are known. Probably the best way to construct this parabola is to draw it graphically through the three points

$$x_1 = -l, \qquad y_1 = y_0(e_0 - e_1 + e_2),$$
$$x_2 = 0, \qquad y_2 = y_0 e_0,$$
$$x_3 = l, \qquad y_3 = y_0(e_0 + e_1 + e_2).$$

Case (iii). *To fit a parabola of the third order to a series of observations.*

Let the curve be

$$y = y_0\left\{e_0 + e_1\frac{x}{l} + e_2\left(\frac{x}{l}\right)^2 + e_3\left(\frac{x}{l}\right)^3\right\}.$$

The equations to find the e's will now be

$$\lambda_0 = e_0 + \tfrac{1}{3}e_2, \qquad \lambda_1 = \tfrac{1}{3}e_1 + \tfrac{1}{5}e_3,$$
$$\lambda_2 = \tfrac{1}{3}e_0 + \tfrac{1}{5}e_2, \qquad \lambda_3 = \tfrac{1}{5}e_1 + \tfrac{1}{7}e_3.$$

Hence
$$e_0 = \tfrac{9}{4}(3\lambda_0 - 5\lambda_2), \qquad e_1 = \tfrac{15}{4}(5\lambda_1 - 7\lambda_3),$$
$$e_2 = \tfrac{15}{4}(3\lambda_2 - \lambda_0), \qquad e_3 = \tfrac{35}{4}(-3\lambda_1 + 5\lambda_3).$$

Case (iv). *To fit a parabola of the fourth order to a series of observations.*

Let the curve be

$$y = y_0\left\{e_0 + e_1\frac{x}{l} + e_2\left(\frac{x}{l}\right)^2 + e_3\left(\frac{x}{l}\right)^3 + e_4\left(\frac{x}{l}\right)^4\right\}.$$

Then
$$\lambda_0 = e_0 + \tfrac{1}{3}e_2 + \tfrac{1}{5}e_4, \qquad \lambda_1 = \tfrac{1}{3}e_1 + \tfrac{1}{5}e_3,$$
$$\lambda_2 = \tfrac{1}{3}e_0 + \tfrac{1}{5}e_2 + \tfrac{1}{7}e_4, \qquad \lambda_3 = \tfrac{1}{5}e_1 + \tfrac{1}{7}e_3,$$
$$\lambda_4 = \tfrac{1}{5}e_0 + \tfrac{1}{7}e_2 + \tfrac{1}{9}e_4.$$

Hence we find

$$e_0 = \tfrac{175}{64}(15\lambda_0 - 70\lambda_2 + 63\lambda_4), \qquad e_1 = \tfrac{15}{4}(5\lambda_1 - 7\lambda_3),$$
$$e_2 = \tfrac{1575}{32}(-5\lambda_0 + 42\lambda_2 - 45\lambda_4), \qquad e_3 = \tfrac{35}{4}(-3\lambda_1 + 5\lambda_3),$$
$$e_4 = \tfrac{3465}{64}(3\lambda_0 - 30\lambda_2 + 35\lambda_4).$$

Case (v). *To fit a parabola of the fifth order to a series of observations.*

Let the curve be

$$y = y_0\left\{e_0 + e_1\frac{x}{l} + e_2\left(\frac{x}{l}\right)^2 + e_3\left(\frac{x}{l}\right)^3 + e_4\left(\frac{x}{l}\right)^4 + e_5\left(\frac{x}{l}\right)^5\right\}.$$

Then
$$\lambda_0 = e_0 + \tfrac{1}{3}e_2 + \tfrac{1}{5}e_4, \qquad \lambda_1 = \tfrac{1}{3}e_1 + \tfrac{1}{5}e_3 + \tfrac{1}{7}e_5,$$
$$\lambda_2 = \tfrac{1}{3}e_0 + \tfrac{1}{5}e_2 + \tfrac{1}{7}e_4, \qquad \lambda_3 = \tfrac{1}{5}e_1 + \tfrac{1}{7}e_3 + \tfrac{1}{9}e_5,$$
$$\lambda_4 = \tfrac{1}{5}e_0 + \tfrac{1}{7}e_2 + \tfrac{1}{9}e_4, \qquad \lambda_5 = \tfrac{1}{7}e_1 + \tfrac{1}{9}e_3 + \tfrac{1}{11}e_5.$$

Hence we have

$$e_0 = \tfrac{15}{64}\ (15\lambda_0 - 70\lambda_2 + 63\lambda_4), \qquad e_1 = \tfrac{105}{64}\ (35\lambda_1 - 126\lambda_3 + 99\lambda_5),$$
$$e_2 = \tfrac{105}{32}(-5\lambda_0 + 42\lambda_2 - 45\lambda_4), \qquad e_3 = \tfrac{315}{32}(-21\lambda_1 + 90\lambda_3 - 77\lambda_5),$$
$$e_4 = \tfrac{315}{64}\ (3\lambda_0 - 30\lambda_2 + 35\lambda_4), \qquad e_5 = \tfrac{693}{64}\ (15\lambda_1 - 70\lambda_3 + 63\lambda_5).$$

Case (vi). *To fit a parabola of the sixth order to a series of observations.*

Let the curve be

$$y = y_0\left\{e_0 + e_1\frac{x}{l} + e_2\left(\frac{x}{l}\right)^2 + e_3\left(\frac{x}{l}\right)^3 + e_4\left(\frac{x}{l}\right)^4 + e_5\left(\frac{x}{l}\right)^5 + e_6\left(\frac{x}{l}\right)^6\right\}.$$

Then

$$\lambda_0 = e_0 + \tfrac{1}{3}e_2 + \tfrac{1}{5}e_4 + \tfrac{1}{7}e_6, \qquad \lambda_1 = \tfrac{1}{3}e_1 + \tfrac{1}{5}e_3 + \tfrac{1}{7}e_5,$$
$$\lambda_2 = \tfrac{1}{3}e_0 + \tfrac{1}{5}e_2 + \tfrac{1}{7}e_4 + \tfrac{1}{9}e_6, \qquad \lambda_3 = \tfrac{1}{5}e_1 + \tfrac{1}{7}e_3 + \tfrac{1}{9}e_5,$$
$$\lambda_4 = \tfrac{1}{5}e_0 + \tfrac{1}{7}e_2 + \tfrac{1}{9}e_4 + \tfrac{1}{11}e_6, \qquad \lambda_5 = \tfrac{1}{7}e_1 + \tfrac{1}{9}e_3 + \tfrac{1}{11}e_5,$$
$$\lambda_6 = \tfrac{1}{7}e_0 + \tfrac{1}{9}e_2 + \tfrac{1}{11}e_4 + \tfrac{1}{13}e_6.$$

Hence we find

$$e_0 = \tfrac{35}{256}\ (35\lambda_0 - 315\lambda_2 + 693\lambda_4 - 429\lambda_6), \qquad e_1 = \tfrac{105}{64}\ (35\lambda_1 - 126\lambda_3 + 99\lambda_5),$$
$$e_2 = \tfrac{315}{256}(-35\lambda_0 + 567\lambda_2 - 1485\lambda_4 + 1001\lambda_6), \qquad e_3 = \tfrac{315}{32}(-21\lambda_1 + 90\lambda_3 - 77\lambda_5),$$
$$e_4 = \tfrac{3465}{256}\ (7\lambda_0 - 135\lambda_2 + 385\lambda_4 - 273\lambda_6), \qquad e_5 = \tfrac{693}{64}\ (15\lambda_1 - 70\lambda_3 + 63\lambda_5),$$
$$e_6 = \tfrac{3003}{256}(-5\lambda_0 + 105\lambda_2 - 315\lambda_4 + 231\lambda_6).$$

(13) *Illustration VI.* In order to thoroughly test the manner in which successive parabolas fit more and more closely to a series of observations, I have taken as a first illustration a very unpromising series of observations given by Thiele in his *Forelaesninger over Almindelig Iagttagelseslaere* (Kjøbenhavn, 1889), p. 12. I say *unpromising* because the observations are not such as one would in practice attempt to fit with a parabolic curve; they form a frequency distribution for which my skew frequency curve of limited range gives a very good fit as we have seen above in § 6. But to fit these unpromising observations even approximately is of great interest; the process shows us much more clearly than would otherwise be the case the struggles of the successive parabolas to get their points of inflexion to the approximately correct positions, and, to speak anthropomorphically, their rather futile attempts to bend themselves into the shape of the observation curve. But a still more important principle is illustrated when we compare these parabolas with the generalised frequency curve, namely that the number of constants at our disposal is no measure of the goodness of the fit. The skew frequency curve with three constants fits much better than the parabola of the sixth order with seven constants. Thus in fitting an empirical curve to observations it is all-important to make a suitable choice of that curve; i.e. to determine whether it should be algebraical, exponential, trigonometrical, etc. There is indeed very little to justify the readiness with which in practice a parabola of one or another order is selected to describe the results of observation.

A little consideration will frequently lead to the selection of a curve with as few or even fewer constants giving a far better fit.

Thiele's observations are given in § 6. We shall attempt to fit a parabola-series to Thiele's trapezoidal polygon between $x=6$ and $x=20$, i.e. we shall take $2l=14$.

The origin for moments was taken at $x=13$, and the successive moments $m_0\mu_1'$, $m_0\mu_2'$, $m_0\mu_3'$, $m_0\mu_4'$, $m_0\mu_5'$ and $m_0\mu_6'$ calculated for the system of trapezia from the concentrated y's by the formulæ

$$\mu_1' = \nu_1', \qquad \mu_4' = \nu_4' + \nu_2' c^2 + \tfrac{1}{15} c^4,$$
$$\mu_2' = \nu_2' + \tfrac{1}{6} c^2, \qquad \mu_5' = \nu_5' + \tfrac{5}{3}\nu_3' c^2 + \tfrac{1}{3}\nu_1' c^4,$$
$$\mu_3' = \nu_3' + \tfrac{1}{2}\nu_1' c^2, \qquad \mu_6' = \nu_6' + \tfrac{5}{2}\nu_4' c^2 + \nu_2' c^4 + \tfrac{1}{28} c^6,$$

where c is the base element, or in our case unity, and

$$\nu_s' = S\{y(x-13)^s\}.$$

These formulæ are deduced in my memoir on "Skew Variation in Homogeneous Material," *Phil. Trans.* Vol. 186, A, p. 349*. See also § 10 above.

Then $\lambda_s = \dfrac{\mu_s'}{l^s}$ was calculated, and the following values obtained:

$$\lambda_1 = -\cdot 162{,}857, \qquad \lambda_2 = \cdot 114{,}748,$$
$$\lambda_3 = -\cdot 033{,}778, \qquad \lambda_4 = \cdot 030{,}712,$$
$$\lambda_5 = -\cdot 010{,}204, \qquad \lambda_6 = \cdot 012{,}141.$$

In addition we have $y_0 = 35{\cdot}7143$,

Mean $x = 11{\cdot}86$.

From these values the e's were calculated and the following series of parabolas obtained, x being measured from the mid-range:

(i) $y = 35{\cdot}7143\{1 - \cdot 488{,}571\,(x/l)\}$,

(ii) $y = 35{\cdot}7143\{1{\cdot}819{,}694 - \cdot 488{,}571\,(x/l) - 2{\cdot}459{,}082\,(x/l)^2\}$,

(iii) $y = 35{\cdot}7143\{1{\cdot}819{,}694 - 2{\cdot}166{,}885\,(x/l) - 2{\cdot}459{,}082\,(x/l)^2 + 2{\cdot}797{,}191\,(x/l)^3\}$,

(iv) $y = 35{\cdot}7143\{2{\cdot}086{,}513 - 2{\cdot}166{,}885\,(x/l) - 5{\cdot}127{,}275\,(x/l)^2 + 2{\cdot}797{,}191\,(x/l)^3 + 3{\cdot}112{,}892\,(x/l)^4\}$,

(v) $y = 35{\cdot}7143\{2{\cdot}086{,}513 - 4{\cdot}026{,}295\,(x/l) - 5{\cdot}127{,}275\,(x/l)^2 + 11{\cdot}474{,}432\,(x/l)^3 + 3{\cdot}112{,}892\,(x/l)^4 - 7{\cdot}809{,}518\,(x/l)^5\}$,

(vi) $y = 35{\cdot}7143\{2{\cdot}041{,}057 - 4{\cdot}026{,}295\,(x/l) - 4{\cdot}172{,}701\,(x/l)^2 + 11{\cdot}474{,}432\,(x/l)^3 + \cdot 249{,}170\,(x/l)^4 - 7{\cdot}809{,}518\,(x/l)^5 + 2{\cdot}100{,}062\,(x/l)^6\}$.

* These are not the proper formulæ if we considered Thiele's observations as the areas of a frequency curve, but what we are here doing is to fit a series of curves as closely as possible to a polygonal area.

The ordinates for these curves corresponding to the original observations, i.e. $\pm x/l = 0,\ 1/7,\ 2/7,\ 3/7,\ 4/7,\ 5/7,\ 6/7,\ 1$, were ascertained, and are given in the accompanying Table. The curves are plotted in Fig. 9.

Ordinates of Thiele's Observations.

Actual Numbers		Skew Frequency	Parabolas					
			1st	2nd	3rd	4th	5th	6th
6	*0*	·06	18·3	−40·3	− ·3	25·1	−10·3	20·7
7	*3*	1·89	20·8	−14·5	−3·0	−3·4	5·8	−7·8
8	*7*	13·55	23·25	7·7	1·3	−8·8	4·8	14·4
9	*35*	42·26	25·7	26·3	10·7	1·0	3·9	50·5
10	*101*	77·29	28·2	41·4	23·6	19·3	11·2	79·7
11	*89*	97·49	30·7	52·8	38·0	40·5	28·25	92·9
12	*9¼*	93·54	33·2	60·7	52·4	60·1	51·5	89·2
13	*70*	72·76	35·7	65·0	65·0	74·5	74·5	72·9
14	*46*	47·94	38·2	65·7	74·0	81·6	90·2	50·5
15	*30*	27·60	40·7	62·8	77·6	80·1	92·4	28·8
16	*15*	14·21	43·2	56·3	74·2	69·9	78·0	12·9
17	*4*	6·46	45·7	46·3	61·9	52·2	49·3	5·1
18	*5*	2·88	48·2	32·6	39·05	28·9	15·3	3·9
19	*1*	1·16	50·7	15·4	3·9	3·4	−5·8	3·7
20	*0*	·44	53·2	−5·4	−45·3	−19·9	15·5	−5·1

Taking the 6th parabola as the best let us compare the results found from it with those obtained from the skew curve. Let Δ_1 be the difference from observation in the latter case, Δ_2 in the former. We find

Δ_1	Δ_2	Δ_1	Δ_2
− ·1	−20·7	− 1·9	− 4·5
+ 1·1	+10·8	+ 2·4	+ 1·2
− 6·6	− 7·4	+ ·8	+ 2·1
− 7·3	−15·5	− 2·5	− 1·1
+23·7	+21·3	+ 2·1	+ 1·1
− 8·5	− 3·9	− ·2	− 2·7
+ ·5	+ 4·8	− ·4	+ 5·1
− 2·8	− 2·9		

Now whether we measure the goodness of fit by the mean Δ without regard to sign, by the mean square error, or by the value of $S(\Delta^2/y)$, we reach the same result,—there is an overwhelming balance in favour of the exponential curve over the algebraic curve. We can make as Thiele* actually does a curve of factorials or even a parabola of the 15th order to go through all the 15 observations, but although we shall thus of course get a better fit than by using a three-constant

* *Forelaesninger over Almindelig Iagttagelseslaere* (Kjøbenhavn, 1889), p. 12.

curve, the fit is purely artificial: a little beyond the range of the actual observations, the parabola will diverge immensely from the sort of result that we could possibly reach if we multiplied our observations so as to get, say, the frequency at $x = 5$, or $x = 23$. The representation is artificial, the parabolic curve cannot give the limited range, or the high contact at its terminals,—well-known characters of such frequency distributions—which can be provided by other curves

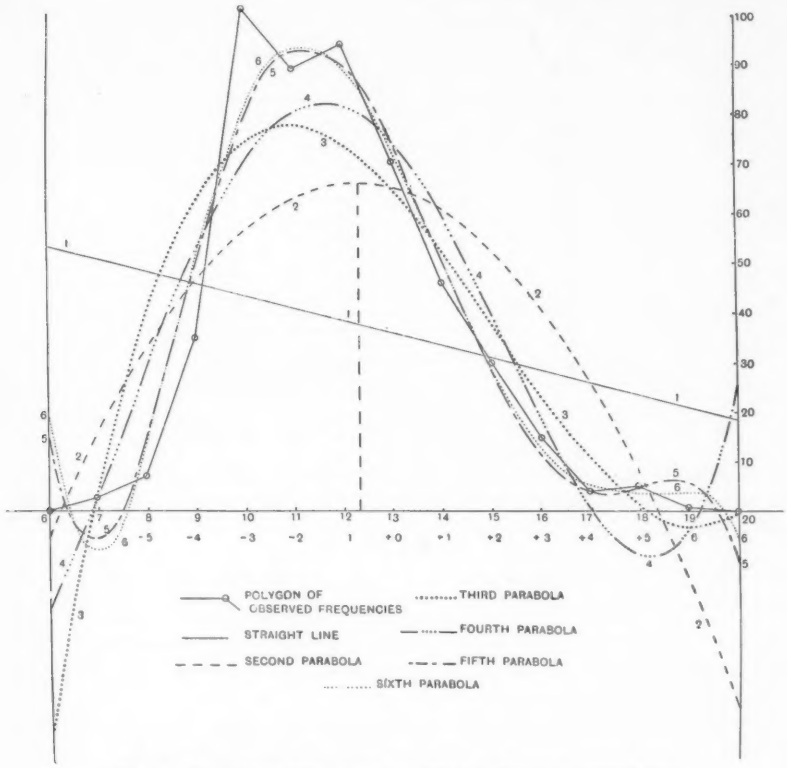


FIG. 9. Example of fitting parabolas (Thiele's frequency observations).

with, perhaps, a quarter the number of constants. Hence the sort of statement so frequently heard,—"Yes, of course, more constants make better fits,"—is only a half truth, and the manner in which engineers, physicists and actuaries so readily use parabolic curves is open to considerable criticism. There are often considerations, lying outside the actual data, which suffice to indicate that trigonometrical, exponential or other types of curves will give better results than parabolas. A parabola which passes even through all the observations may indeed

be a most undesirable representation of the facts, for it has twisted and curled to account for error as well as to give the general sweep of the observations.

(14) *Illustration VII.* As a second illustration of parabola-fitting I will take from the Italian marriage statistics of Perozzo the modal ages of bridegrooms of brides of given ages. With some simple interpolations I have determined these from Perozzo's tables* as approximately the following:

Age of Bride	Probable Age of Groom	Age of Bride	Probable Age of Groom
15·5	25	31·5	33
16·5	25·2	32·5	33·5
17·5	25·4	33·5	34
18·5	25·5	34·5	34·5
19·5	25·5	35·5	36
20·5	25·5	36·5	37
21·5	25·75	37·5	38 5
22·5	26	38·5	39·5
23·5	26	39·5	41·5
24·5	26·8	40·5	41·5
25·5	27	41·5	42·5
26·5	27·5	42·5	43·5
27·5	28	43·5	43·5
28·5	29	44·5	43·5
29·5	30	45·5	43·5
30·5	32		

Now let us take 43·5 as the origin of age for the man, and 30·5 as that for the woman; then if y be the man's age and x the woman's, we have the following series of points:

$x =$ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,

$y =$ 11·5, 10·5, 10, 9·5, 9, 7·5, 6·5, 5, 4, 2, 2, 1, 0, 0, 0, 0,

$x =$ −1, −2, −3, −4, −5, −6, −7, −8, −9, −10, −11, −12, −13, −14, −15,

$y =$ 13·5, 14·5, 15·5, 16, 16·5, 16·7, 17·5, 17·5, 17·75, 18, 18, 18, 18·1, 18·3, 18·5.

The range is thus $2l = 30$, and concentrating and then using the corrective formulæ for trapezoidal polygons given in § 10 we easily find

$$y_0 = 10{\cdot}7867,$$

$$\lambda_1 = -{\cdot}348{,}5000, \qquad \lambda_4 = {\cdot}173{,}6922,$$

$$\lambda_2 = {\cdot}299{,}9564, \qquad \lambda_5 = -{\cdot}137{,}7881,$$

$$\lambda_3 = -{\cdot}198{,}1573, \qquad \lambda_6 = {\cdot}123{,}0257.$$

The e's were then determined from the λ's by the formulæ of § 12 and we have for the series of parabolas, if $l = 15$ years:

* *Reale Accademia dei Lincei.* Anno CCLXXIX, 1882. Nuove Applicazioni del Calcolo delle Probabilità.

1st parabola:

$$y = 10{\cdot}7867\left\{1 - 1{\cdot}0455\left(\frac{x}{l}\right)\right\}.$$

2nd parabola:

$$y = 10{\cdot}7867\left\{1{\cdot}125{,}1635 - 1{\cdot}0455\left(\frac{x}{l}\right) - {\cdot}375{,}4905\left(\frac{x}{l}\right)^2\right\}.$$

3rd parabola:

$$y = 10{\cdot}7867\left\{1{\cdot}125{,}1635 - 1{\cdot}332{,}7459\left(\frac{x}{l}\right) - {\cdot}375{,}4905\left(\frac{x}{l}\right)^2 + {\cdot}478{,}7431\left(\frac{x}{l}\right)^3\right\}.$$

4th parabola:

$$y = 10{\cdot}7867\left\{1{\cdot}159{,}1392 - 1{\cdot}332{,}7459\left(\frac{x}{l}\right) - {\cdot}715{,}2475\left(\frac{x}{l}\right)^2 + {\cdot}478{,}7431\left(\frac{x}{l}\right)^3 + {\cdot}396{,}3832\left(\frac{x}{l}\right)^4\right\}.$$

5th parabola:

$$y = 10{\cdot}7867\left\{1{\cdot}159{,}1392 - 1{\cdot}428{,}4973\left(\frac{x}{l}\right) - {\cdot}715{,}2475\left(\frac{x}{l}\right)^2 + {\cdot}925{,}5752\left(\frac{x}{l}\right)^3 + {\cdot}396{,}3832\left(\frac{x}{l}\right)^4 - {\cdot}402{,}1555\left(\frac{x}{l}\right)^5\right\}.$$

6th parabola:

$$y = 10{\cdot}7867\left\{1{\cdot}108{,}0239 - 1{\cdot}428{,}4973\left(\frac{x}{l}\right) + {\cdot}358{,}1736\left(\frac{x}{l}\right)^2 + {\cdot}925{,}5752\left(\frac{x}{l}\right)^3 - 2{\cdot}823{,}8816\left(\frac{x}{l}\right)^4 - {\cdot}402{,}1555\left(\frac{x}{l}\right)^5 + 2{\cdot}361{,}5275\left(\frac{x}{l}\right)^6\right\}.$$

The ordinates of these six curves were then calculated for the 31 values of x, and the curves themselves with the observations plotted in Fig. 10. We have again an instructive graphical representation of the closer and closer approach of a series of parabolas to a fairly smooth system of observations. It will be seen that the parabola of the 4th order gives quite an excellent representation of the observations, better indeed than the 6th parabola which has too many points of inflexion to dispose of.

(15) With this illustration I close my discussion for the present of curve-fitting. I have endeavoured to show:

(1) that the method of moments must theoretically give good fits;

(2) that it provides a systematic method of fitting a great variety of curves;

(3) that it is over and over again available when the method of least squares fails, or can only be applied with excessive labour;

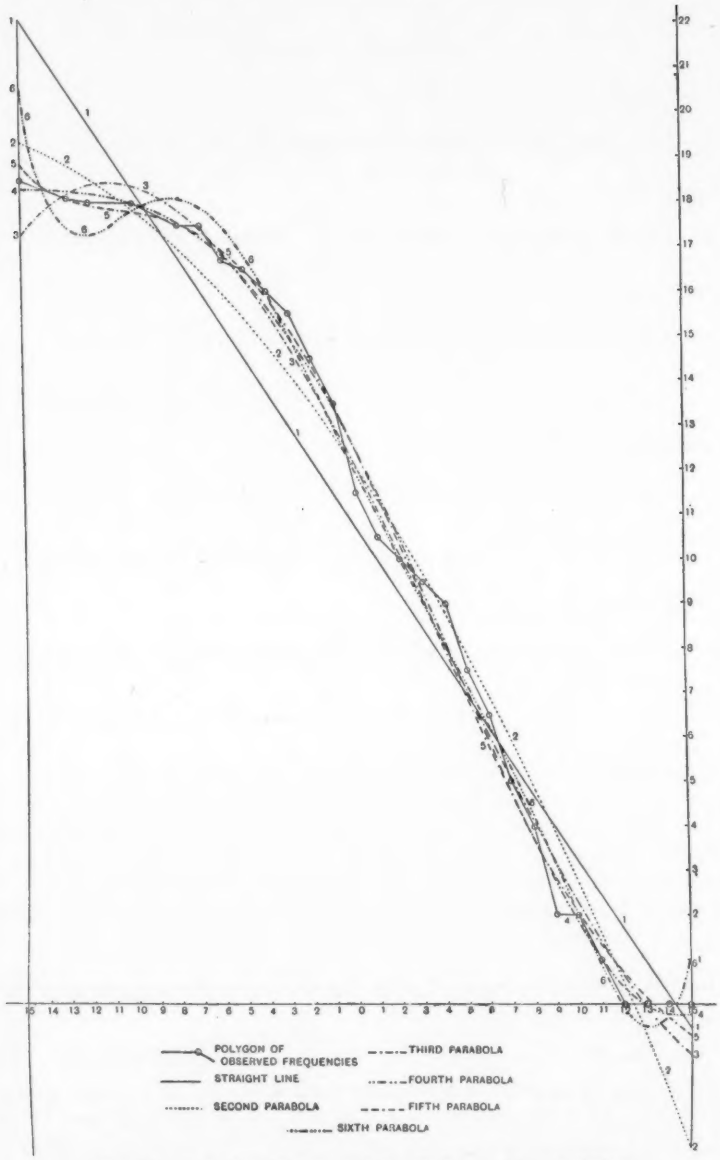


FIG. 10. Example of fitting parabolas (Italian marriage statistics).

(4) that in all cases much depends on the method by which the successive moments are calculated, and that frequency observations and physical measurements require to some extent different processes;

(5) that with a good moment formula the method of moments will give for practical purposes as good fits as the method of least squares when both are applicable;

(6) that for fitting parabolas the method of moments has theoretically the same basis as that of least squares, but in application it is much easier;

and lastly:

(7) that far more really depends for goodness of fit on the equation to the curve selected than on the number of its constants, and that the multiplication of constants to improve fit is not only theoretically undesirable, but does not necessarily lead to the required result.

QUANTITATIVE STUDY OF THE EFFECT OF ENVIRONMENT UPON THE FORMS OF NASSA OBSOLETA AND NASSA TRIVITTATA FROM COLD SPRING HARBOR, LONG ISLAND.

By ABIGAIL CAMP DIMON.

(1) *Introductory.* The aims of this paper are to make a quantitative inquiry into the effect of diverse environmental conditions upon the form of two gastropod species, *Nassa obsoleta* and *Nassa trivittata*, from Cold Spring Harbor, Long Island, and also to record the characteristics of their shells and thus to determine the "place mode" for these shells in that locality. The characters selected for measurement were those described in systematic works as distinguishing the species as far as those characteristics could be easily determined quantitatively. The work was done under the general direction of Dr Charles B. Davenport.

(2) *General Description of Nassa, its range and habits.* *Nassa* is a genus of prosobranch gastropods containing many species distributed over the whole world, chiefly in shallow water. The individuals are usually small, with an ovate shell and a large foot, which is notched behind and carries a horny operculum. The two species, *Nassa obsoleta*, Say (*Ilyanassa obsoleta*, Stimpson; *Buccinum obsoletum*, Gould), and *Nassa trivittata*, Say (*Tritia trivittata*, Adams; *Buccinum trivittatum*, Gould), found commonly at Cold Spring Harbor, are American forms, and with the less common *Nassa vibex* constitute the only recognised littoral species of the genus found on the middle Atlantic coast of the United States. Both species range from the Gulf of St Lawrence to Florida. Verrill ('73) reports *N. trivittata* as abundant at Casco Bay, Maine, and in Vineyard Sound and Buzzard's Bay, and as common along Long Island Sound; whereas *N. obsoleta* he reports as very abundant south of Cape Cod and more local further north. *N. trivittata*, therefore, reaches its maximum numbers further north than *N. obsoleta*. Geologically, *N. trivittata* is older than *N. obsoleta*, having been found in the Miocene of Maryland, Virginia and South Carolina, while *N. obsoleta* has not been reported from further back than the Pliocene.

Comparing in detail the habitats of the two species, it is seen that *N. obsoleta* lives in large numbers on flats and shores left bare by the tide during part of the day, and is not found at any considerable depth; while *N. trivittata* is found in some places at a depth of forty fathoms, and is not found above low water line. Verrill ('73) states that about Vineyard Sound *N. obsoleta* occurs in bays and sounds on sandy and muddy shores and bottoms, and on submerged wood-work, such as the piles of wharves, but that it does not occur on rocky shores and bottoms. *N. obsoleta* was found in brackish water on sand, mud, oyster-beds, eel-grass, and submerged wood-work; but it was not found in the open ocean. *N. trivittata* was found by Verrill in all the habitats of *N. obsoleta* except the muddy shores of the bays and sounds and the muddy shores and bottoms, oyster-beds and eel-grass in the brackish water; it was found in the open ocean on shores and bottoms. With these observations my own experience with the distribution of the two mollusca at Cold Spring Harbor and elsewhere is in complete accord.

The breeding season of *N. obsoleta* is given by Mead ('98) for Woods Holl as the latter part of April. In July and August when nearly all my collecting was done there would often be found in pools left above low water mark large numbers of very small individuals which were evidently the brood of that year. The older snails, however, could not be separated into broods of different ages on the basis either of size or the number of whorls, so that either the growth after the first year is extremely slow or else the snails do not live until the third summer.

(3) *Localities from which the shells were collected.* At Cold Spring Harbor we have, within a small district, several localities in which *Nassa* may be found. The Harbor is a branch of Long Island Sound, five miles long by one mile wide. It is divided into an inner and an outer harbor by a sand-spit that extends nearly across it at about half a mile from its head. The inner harbor is fed by a stream which makes its water decidedly brackish, especially at the surface. For near the mouth of the creek, at the surface, the density may be as low as 1·006 while at the bottom it is 1·016. Under other conditions of wind and tide the density will be about 1·016 throughout. The average height of tide is about 7·5 feet in the inner harbor, and at low tide about half the surface is left bare, exposing flats of black mud on which *Ulva* grows in abundance. In the outer harbor the density of the water is from 1·018 to 1·020. The bottom of the outer harbor consists of mud, oyster-beds or sand, with a good deal of eel-grass; the shores are sandy or muddy with a greater slope than those of the inner harbor, so that there are no extended mud-flats exposed at low tide. At the mouth of the harbor, on the east, the shores are sandy and gravelly and have a considerable slope, and the situation is far less sheltered than within the harbor.

The *Nassa* measured were collected from the three localities marked 1, 2, and 3 on the map, Figure 1. Those from 1 (Laboratory Dock) were gathered from the mud-flats at low tide; those from 2 (Laurelton Dock) were taken from the sandy beach at low tide or dredged from a few feet of water at the same locality; and those from 3 (Lloyd Point) were collected from pools left on the sand beach

at low tide. It was not possible to find *N. trivittata* in abundance in all three localities, so this species was taken only from 3, where the shells were washed up on the beach in considerable numbers and easily collected. Only one or two specimens were dredged from locality 2, and no specimen has been found in the inner harbor.

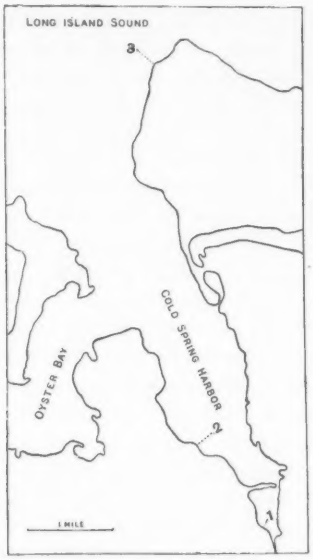


FIG. 1. Sketch Map of Cold Spring Harbor.

(4) *Qualitative comparison of the shells from the different localities.* If handfuls of *N. obsoleta* from each of the three localities marked on the map be compared a decided difference in size will be at once noticed. The shells from 3, the most exposed locality, are much the largest. On the other hand they are much more numerous in the inner harbor. In comparing *N. trivittata* with *N. obsoleta* it is seen that the shells of *N. trivittata* are not covered with algae and are not eroded, while the shells of *N. obsoleta* are covered by algae and much eroded at tip, probably in consequence. Also *N. trivittata* is almost white, whereas *N. obsoleta* is blackish purple, its apical angle is more acute, and its shell is smaller.

(5) *Method of measuring.* The characteristics of the shells of which it was sought to get a quantitative expression were size, shape, number of whorls, color, and roughness of surface. To ensure accuracy the dimensions of the shell and the angle at its apex were all measured two or three times. On account of the length of the process or the great effect of a previous reading on the judgment

the determinations of the number of whorls, color, and roughness were made only once, so these determinations are less accurate. To measure length a micrometer gauge reading to hundredths was used. In Figure 2 A, the line *AD* represents the direction in which length was measured, *B* the direction for diameter, and *C* for greatest length of aperture. The shape of the shell was given by the angle at the apex and the ratios of the diameter to the length and of the aperture to the length. The ratios were calculated from the measurements; the angle was measured directly by means of a bevel protractor. The erosion of the apex

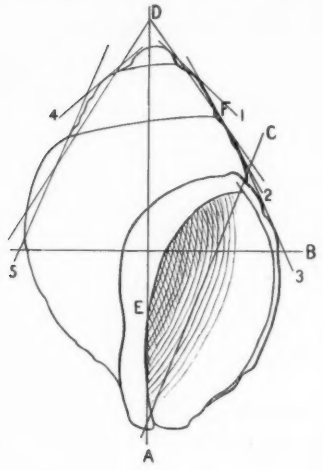


FIG. 2 A.

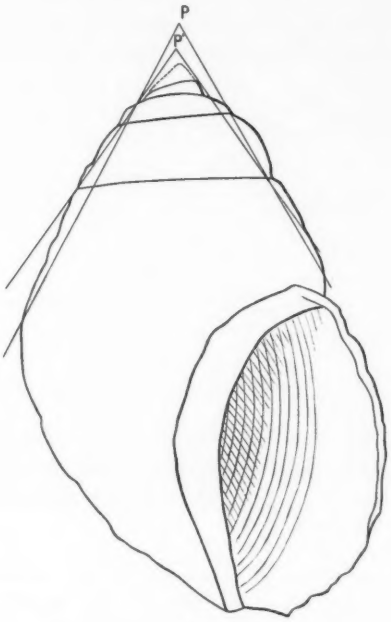


FIG. 2 B.

disturbed the measurement of the angle in the case of *N. obsoleta*, and it was finally decided to take an angle the direction of each side of which would be a compromise between the directions of lines drawn between the center points of successive whorls. *D*, Figure 2 A, shows such a compromise between lines 1, 2, 3, on one side, and 4, 5, on the other, and represents what I have called the apical angle. Even after the more eroded shells had been discarded, the bluntness of the apex often affected the angle, the general tendency being to read it too large, as shown in Figure 2 B. The mean angles of shells from the different localities are therefore to be compared only with caution.

The number of whorls was counted directly. Color was measured by the color top in the region *E*, Figure 2 A; the results are, however, rather rough. The depth of the suture between the whorls was chosen as an approximate measure of roughness. It was measured between the last two whorls, near *F*, Figure 2 A, by laying the shell so that it rested on its aperture and making a camera drawing magnified eight or nine times of the part of the line it was desired to measure. Then the depth of the depression from a tangent drawn to the adjoining elevations was measured on the drawing, and this distance was divided by the magnification of the figure. The dimension thus obtained depends, however, not solely upon the roughness of the shell but also upon the convexity of the individual whorls. Figure 3 shows some types of outlines obtained in the manner described; *b* represents an extreme case of a smooth shell with a deep suture.

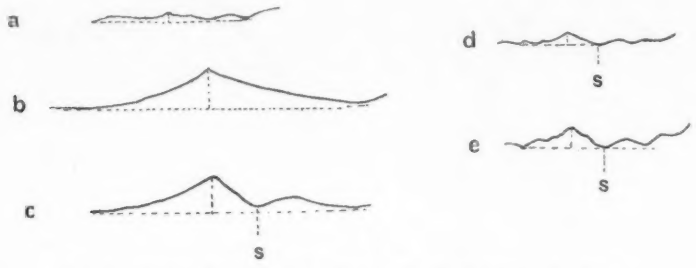


FIG. 3. Camera drawings illustrating the depth of suture. S=shoulder.

(6) *Table of constants of the curves.* In the following table some of the results of the quantitative determinations are given. 1, 2, and 3 represent lots of *N. obsoleta* from the localities referred to; 3*a* is the lot of *N. trivittata* from Lloyd Point.

Discussion of constants. Means. Setting aside for the present *N. trivittata*, a comparison of the means of *N. obsoleta* from the three localities brings out certain points. The shells from 3, the mouth of the harbor, are much longer than the others; those from 1, the inner basin, are the most globose, and have the largest aperture in proportion to their size, and the fewest whorls. In depth of suture the shells from the mouth of the harbor are extreme. The probable error of the color measurement is so great that no stress can be laid on the slight differences recorded. As to the apical angle, the effect of erosion at the apex upon this reading is, as already stated, so considerable that lot 1 (the most eroded) is not strictly comparable with the other lots.

If we correlate these differences with the environmental differences noted above for the three localities, we find the individuals living in less dense water—in the inner harbor—are smaller, more globose, with a larger aperture in proportion

to their length, and a shallower suture. Though the series is not in each case a regular one, with the shells from 1 and 3 at the extremities and those from 2 intermediate, yet the general relationship holds. The tendency of the shells from 2 toward an extreme position is not necessarily inconsistent. Cooke ('95)

TABLE I.

Quantitative data. Table of Constants of the Curves.

	Mean (M)	Probable Error of M	Observed Mode	Theoretical Mode	Standard Deviation (σ)	Probable Error of σ	Coefficient of Variability	Type	Skewness
Length: *1*	13·92 mm.	±·0336 mm.	13·50 mm.	13·79 mm.	1·120 mm.	±·0238 mm.	7·86 %	IV	+·1206
2	12·75 „	±·0372 „	13·00 „	12·76 „	1·232 „	±·0263 „	9·10 %	IV	−·0051
3	17·14 „	±·0664 „	17·00 „	16·46 „	1·890 „	±·0469 „	11·03 %	I	+·3590
3a	11·96 „	±·0814 „	12·00 „	12·30 „	2·590 „	±·0575 „	21·66 %	I	−·1328
Diameter / Length: *1*	59·74 %	±·109 %	57·50 %	59·09 %	3·614 %	±·0773 %	6·05 %	I	+·1796
2	58·30 %	±·103 %	58·50 %	58·37 %	3·416 %	±·0760 %	5·86 %	IV	−·0216
3	58·71 %	±·099 %	59·00 %	55·56 %	2·825 %	±·0714 %	4·82 %	IV	+·0608
3a	52·14 %	±·118 %	51·50 %	51·30 %	3·740 %	±·0835 %	7·17 %	IV	+·2238
Aperture / Length: *1*	64·12 %	±·107 %	64·50 %	64·25 %	3·522 %	±·0754 %	5·49 %	IV	−·0354
2	62·32 %	±·080 %	62·50 %	62·63 %	2·646 %	±·0565 %	4·25 %	IV	−·1178
3	62·71 %	±·092 %	63·00 %	62·84 %	2·626 %	±·0652 %	4·19 %	IV	−·0481
3a	51·24 %	±·086 %	49·00 %	50·87 %	2·727 %	±·0608 %	5·32 %	IV	+·1350
Apical angle: *1*	53·44°	±·176°	57·00°	53·93°	5·832°	±·1241°	10·91 %	I	−·0844
2	55·46°	±·151°	57·00°	55·79°	5·046°	±·1071°	9·10 %	I	−·0659
3	53·09°	±·200°	53·00°	53·30°	5·702°	±·1416°	10·74 %	I	−·0361
3a	47·48°	±·129°	48·00°	47·17°	4·102°	±·0910°	8·64 %	I	+·0766
Number of whorls: *1*	4·88	±·017	5·00	4·63	·560	±·0120	11·48 %	I	+·3790
2	5·49	±·015	5·25	5·43	·496	±·0106	9·03 %	IV	+·1161
3	5·34	±·014	5·38	5·36	·404	±·0102	7·57 %	I	−·0379
3a	7·01	±·019	6·87	7·16	·595	±·0113	8·49 %	IV	−·2480
P. c. of Red in color: *1*	10·44 %	±·063 %	10·00 %	—	2·063 %	±·0447 %	19·76 %	—	—
2	10·09 %	±·076 %	9·50 %	—	2·546 %	±·0542 %	25·17 %	—	—
3	10·51 %	±·065 %	10·00 %	9·20 %	1·860 %	±·0462 %	17·29 %	I	+·7018
P. c. of Black in color: *1*	88·44 %	±·072 %	90·00 %	—	2·336 %	±·0506 %	2·64 %	—	—
2	88·38 %	±·084 %	89·50 %	—	2·798 %	±·0596 %	3·17 %	—	—
3	88·11 %	±·083 %	89·00 %	88·92 %	2·352 %	±·0585 %	2·12 %	I	−·3465
Depth of suture: *1*	25·98 mm./100	±·258 mm./100	24·00 mm./100	22·80 mm./100	8·157 mm./100	±·1822 mm./100	31·40 %	I	+·3895
2	19·83 „	±·194 „	18·50 „	17·61 „	6·423 „	±·1405 „	32·39 %	IV	+·3458
3	33·35 „	±·418 „	28·00 „	27·20 „	11·890 „	±·2956 „	35·65 %	I	+·5177
3a	42·15 „	±·299 „	43·00 „	41·75 „	9·530 „	±·2117 „	22·61 %	IV	+·0420

has noted that where closely related forms live in brackish water and in sea water, the forms in brackish water will in general have a less solid shell than the forms in sea water, yet as the water becomes still more dilute, the forms inhabiting it have more solid shells. Possibly there are conditions of the same kind holding for the size and shape of the shells from the three localities. The stunting influence, for example, may be checked to some degree in the inner harbor, because there the food supply is most abundant.

The effect of density of water on the size and shape of animals living therein has been often noticed. Bronn ('62), in a section on the effect of salt in the water, states that in the Baltic, containing only 4—6 p.m. salt, *Buccinum undatum* and *Littorina litorea* live, but are always small and stunted. In the Mediterranean, containing 37 p.m. salt, oysters are small; at 30—20 p.m. they are, if not largest, of the best flavor; and at 18—17 p.m. they are stunted: which would seem to show an optimum content of salt, above and below which the animals are dwarfed. Simroth ('91) gives as the peculiarity of brackish water fauna that fewer species but more individuals are present; and quotes Möbius and Heincke to the effect that fewer species *of smaller size* are characteristic. Walther ('93) says that the less dense water of the Baltic dwarfs many of the animals living therein, and he gives illustrations among mollusks and worms. Bumpus ('98) in an article on the variations of *Littorina litorea* gives statistics that show that the more dense the water the less globose are the shells of the animals living therein. The means of the curves given by Bumpus in his paper, as calculated by Duncker ('98), express more nearly the condition of the whole population than do Bumpus's own figures, and are therefore used in comparison. The water at Bristol Narrows and Warren River, in Rhode Island, is less dense than the water at Seaconnet and Newport. The globosities are 90·77 °/₀ and 91·08 °/₀ at Bristol Narrows, 92·69 °/₀ at Warren River, 89·72 °/₀ at Seaconnet, and 89·18 °/₀ at Newport; showing decidedly greater globosity in the less dense water. The data as to relative densities of the water in other localities from which Bumpus collected shells are not given, so no further conclusions can be drawn. The foregoing evidence is in accord with the observations noted above on *N. obsoleta*.

If the means of the different characteristics of *N. trivittata* be compared with the means of *N. obsoleta*, the observations that were made qualitatively will be verified. The shells of *N. trivittata* are rougher, smaller, less globose than the shells of *N. obsoleta*, and with a more acute angle at the apex. The range of each character measured overlaps considerably, and even the means are not very far apart.

(7) *Comparison of Cold Spring Harbor type with the normal type.* If we compare the dimensions of the *Nassa* of Cold Spring Harbor with typical measurements given by Apgar ('91) and Gould ('70) for the two species, we find various interesting differences. After looking, by the courtesy of Professor Pilsbury, over the collection of *Nassa* shells from different localities in the museum of the Philadelphia Academy of Sciences, it seemed fair to accept an

average between the dimensions given by Apgar and Gould as representing the normal type of *Nassa*, found in more exposed situations than Cold Spring Harbor. The second and fourth columns of Table II. give this average for *N. obsoleta* and *N. trivittata* respectively, while the first column gives an average of the means of *N. obsoleta* from localities 1, 2, and 3, and the third column gives the means of *N. trivittata* from 3. Table II. shows that *the Cold Spring Harbor shells of N. obsoleta are plainly depauperate,—smaller, more globose, and with a larger aperture and fewer whorls than the normal. All these signs of depauperization may be associated with the lack of density of the water, and the sheltered situation as compared with the open ocean.*

TABLE II.

Comparison of Cold Spring Harbor type of snails with the normal type.

	Cold Spring Harbor *N. obsoleta*	Normal *N. obsoleta*	Lloyd Point *N. trivittata*	Normal *N. trivittata*
Length...	14·6 mm.	25·0 mm.	12·0 mm.	17·7 mm.
Diameter	8·6 „	12·3 „	6·2 „	7·8 „
$\frac{\text{Diameter}}{\text{Length}}$	59 %	49 %	52 %	44 %
$\frac{\text{Aperture}}{\text{Length}}$	63 %	50 %	51 %	40 %
Angle	54°	52·5°	47·5°	45°
Number of Whorls	5·5	6·0	7·0	7·0

This depauperization is evident in the case of *N. trivittata* as well as *N. obsoleta*, and may be explained in the same way. Long Island Sound, as a whole, presents conditions different from those found in the open ocean, its water being less dense on account of the rivers that empty into it, and its situation more sheltered. *N. trivittata*, in fact, is found much more abundantly in the open ocean, and the locality from which the lot measured was collected represents the furthest limit of its intrusion into sheltered, relatively fresh water, in which respect it may be compared with locality 1 as a habitat for *N. obsoleta*. It is what we should expect, therefore, that *the N. trivittata from Cold Spring Harbor show the same relation to normal N. trivittata as the N. obsoleta from locality* 1 *show to normal N. obsoleta.*

(8) *Coefficients of variability.* In considering the coefficients of variability it must be kept in mind that comparisons must be made with caution. Thus, for example, the variability of red is given in Table I. as from 17 % to 25 %, and the variability of black as from 2 % to 3 %. It is obviously incorrect to say that red is from eight to twelve times as variable as black, so that here the coefficient of variability has evidently a different significance in the case of the two colors.

Likewise it cannot be assumed that the coefficient of variability has the same significance for any two characteristics.

If now we run over the coefficients of variability we see that the most variable character is depth of suture, where the variability is about one-third of the depth. It is interesting to observe that this most variable character is to general observation one of the most distinctive of all those measured. That is, it is what Brewster ('99) would call a "conspicuous" character, and to the degree to which it represents roughness of the shell it would have great weight in determining the species. Its great variability is, therefore, in accord with the thesis of Brewster ('97) that "any measurable quality is, in general, variable in individuals in proportion as it is a distinguishing character of the group to which the individuals belong," though the variability is in this case much more extreme than in the case of any evidence Brewster offers.

(9) *Correlations.*

Table III. shows the amount of correlation between some of the characters. The characters of *N. trivittata* show a much greater amount of correlation than those of *N. obsoleta*, and the greatest amount of correlation is shown

TABLE III.

The Amount of Correlation between Various Characteristics.

Locality and Species	First Character	Second Character	Coefficient of Correlation
3. *N. obsoleta* ...	Length	Number of Whorls	·0640
3. *N. trivittata* ...	"	" "	·4178
1. *N. obsoleta* ...	"	" "	·2860
3. "	"	$\frac{\text{Diameter}}{\text{Length}}$	− ·2641
3. *N. trivittata* ...	"	"	− ·7193
3. "	Length	Apical Angle	− ·6577
1. *N. obsoleta* ...	$\frac{\text{Diameter}}{\text{Length}}$	"	·4787
3. *N. trivittata* ...	"	"	·7941
3. *N. obsoleta* ...	Whorls	Angle	·0077

between the angle and the ratio of diameter to length (which we have called globosity). Since globosity is the most important factor in determining the apical angle, it is natural that the correlation between these two characters should be great. The relatively small correlation between length and number of whorls leads us to look for other factors in the determination of length than number of whorls. These factors we find in the size of the whorls and the inclination of the axis of the whorls to the axis of the shells. The values of the correlations between the characters length, index, and apical angle are largely determined, not only in magnitude but in sign, by the fact that, other things being constant, the index

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and apical angle vary *inversely* as the length. Hence "spurious correlation" occurs, and we should *a priori* expect index and angle to be negatively correlated with length and positively correlated with each other. The correlation of diameter and length would have to be determined in order to measure the amount of "spurious correlation" between index and length.

(10) *Description of Abnormal Snail.* About one hundred shells from each locality were measured while the animals inhabiting them were still alive, and of this number one animal was seen with the left tentacle double and bearing two eyes, as represented in Figure 4. All the rest of the specimens were killed before

FIG. 4. Freehand outline sketch of snail from locality 2.

measuring, and as the foot was retracted, the tentacles could not be seen. No other sport of any kind was observed. Bateson ('94) has reported cases of doubling of tentacles and eyes as follows: *Patella vulgata*, tentacle and eye repeated on the left side; *Physa acuta*, right tentacle bifid; three cases of a supernumerary eye without doubling of the tentacle; and one case (*subemarginula*) where both eye-stalks were double, and each bore a supernumerary eye. This condition, then, would seem to be not uncommon, and is mentioned and figured merely as adding one more to the cases of similar abnormality already known.

Summary.

I. There are in Cold Spring Harbor and vicinity two species of *Nassa*, with no forms intermediate in all characters, though their habitats overlap.

II. When compared with the normal types for the species, both these species are depauperate, being smaller, more globose, with a larger aperture in proportion to their length, more obtuse apical angle; and in the case of *N. obsoleta*, fewer whorls. This depauperization is connected with the slight density of the water in Cold Spring Harbor, as compared with the open ocean.

III. If the three lots of *N. obsoleta* be compared with one another, those from within the harbor are seen to be more depauperate than those nearer its mouth. This depauperization is also connected with the density of the water in the three localities from which the specimens were collected.

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APPENDIX A.

Frequency Data. The class is designated both by a serial number and by its middle absolute value expressed, in the case of length in millimeters, in the ratios of diameter to length and aperture to length in percentages, in the case of angle in degrees, of whorls in units of one whorl, of color in percentages, and of depth of suture in units of one-hundredth of a millimeter. 1, 2, 3, and 3*a*, stand, as in Table I., for the lots of *N. obsoleta* shells from localities 1, 2, and 3, and the *N. trivittata* shells from 3, respectively.

Length: Locality 1. Total frequency, 504.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
	10·50	11·0	11·5	12·0	12·5	13·0	13·5	14·0	14·5	15·0	15·5	16·0	16·5	17·0	17·5
Frequency	2	0	4	16	53	65	91	84	82	52	32	9	7	5	2

Length: Locality 2. Total frequency, 499

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*
	7·5	8·0	8·5	9·0	9·5	10·0	10·5	11·0	11·5	12·0	12·5	13·0	13·5	14·0	14·5	15·0	15·5	16·0	16·5	17·0	17·5
Frequency	1	2	0	1	2	2	8	28	42	83	86	89	64	44	22	13	7	3	0	1	1

Length: Locality 3. Total frequency, 368.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	13·0	14·0	15·0	16·0	17·0	18·0	19·0	20·0	21·0	22·0
Frequency	5	19	51	68	78	56	46	31	13	1

Length: Locality 3a. Total frequency, 461.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
	5·0	6·0	7·0	8·0	9·0	10·0	11·0	12·0	13·0	14·0	15·0	16·0	17·0	18·0	19·0
Frequency	2	8	15	17	39	50	55	76	71	46	44	28	8	1	1

$\frac{Diameter}{Length}$: *Locality 1.* Total frequency, 497.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	51·5	53·5	55·5	57·5	59·5	61·5	63·5	65·5	67·5	69·5	71·5
Frequency	6	19	56	117	104	94	54	28	12	6	1

$\frac{Diameter}{Length}$: *Locality 2.* Total frequency, 498.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	46·5	48·5	50·5	52·5	54·5	56·5	58·5	60·5	62·5	64·5	66·5
Frequency	1	0	3	20	50	114	150	100	45	10	5

$\frac{Diameter}{Length}$: *Locality 3.* Total frequency, 369.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
	51·0	52·0	53·0	54·0	55·0	56·0	57·0	58·0	59·0	60·0	61·0	62·0	63·0	64·0	65·0	66·0	67·0
Frequency	1	8	2	10	17	41	45	48	64	39	37	28	15	6	1	3	4

$\frac{Diameter}{Length}$: *Locality 3a.* Total frequency, 456.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
	41·5	43·5	45·5	47·5	49·5	51·5	53·5	55·5	57·5	59·5	61·5	63·5
Frequency	2	1	9	49	103	112	74	48	32	12	12	2

$\frac{Aperture}{Length}$: *Locality 1.* Total frequency, 497.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	12
	52·5	54·5	56·5	58·5	60·5	62·5	64·5	66·5	68·5	70·5	72·5	74·5
Frequency	3	3	8	25	54	115	133	83	42	21	9	1

$\frac{Aperture}{Length}$: *Locality 2.* Total frequency, 499.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	48·5	50·5	52·5	54·5	56·5	58·5	60·5	62·5	64·5	66·5	68·5
Frequency	1	0	1	2	8	42	124	163	116	34	8

$\frac{Aperture}{Length}$: *Locality 3.* Total frequency, 369.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
	54·0	55·0	56·0	57·0	58·0	59·0	60·0	61·0	62·0	63·0	64·0	65·0	66·0	67·0	68·0	69·0	70·0
Frequency	2	1	2	4	8	20	28	51	54	63	54	27	30	12	8	4	1

$\frac{Aperture}{Length}$: *Locality 3a.* Total frequency, 458.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*
	41·0	42·0	43·0	44·0	45·0	46·0	47·0	48·0	49·0	50·0	51·0	52·0	53·0	54·0	55·0	56·0	57·0	58·0	59·0	60·0	61·0
Frequency	1	0	0	2	1	7	15	27	72	71	63	63	51	35	21	14	7	2	1	4	1

Angle: Locality 1. Total frequency, 502.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*
	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71
Frequency	1	3	3	8	12	24	39	48	56	65	65	73	43	35	14	8	4	0	1

Angle: Locality 2. Total frequency, 505.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
	41	43	45	47	49	51	53	55	57	59	61	63	65	67
Frequency	3	5	10	21	31	56	61	73	77	73	50	33	9	3

Angle: Locality 3. Total frequency, 369.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69
Frequency	2	6	15	21	29	38	35	58	52	40	30	26	9	5	2	1

Angle: Locality 3a. Total frequency, 462.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	38	40	42	44	46	48	50	52	54	56	58
Frequency	5	19	35	73	77	94	77	36	31	10	5

Whorls: Locality 1. Total frequency, 493.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	4·00	4·25	4·50	4·75	5·00	5·25	5·50	5·75	6·00	6·25
Frequency	24	75	71	81	110	51	29	23	23	6

Whorls: Locality 2. Total frequency, 494.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	4·25	4·50	4·75	5·00	5·25	5·50	5·75	6·00	6·25	6·50	6·75
Frequency	1	4	11	62	138	102	94	60	15	4	3

Whorls: Locality 3. Total frequency, 357.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	4·13	4·38	4·63	4·88	5·13	5·38	5·63	5·88	6·13	6·38
Frequency	1	7	7	65	61	102	55	48	9	2

Whorls: Locality 3a. Total frequency, 439.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
	4·88	5·13	5·38	5·63	5·88	6·13	6·38	6·63	6·88	7·13	7·38	7·63	7·88	8·13	8·38	8·63
Frequency	2	1	2	6	20	13	35	37	86	84	68	42	36	4	2	1

Redness: Locality 1. Total frequency, 484.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Frequency	11	53	84	150	81	49	23	11	12	2	1	3	2	0	2

Redness: Locality 2. Total frequency, 502.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
	6·5	7·5	8·5	9·5	10·5	11·5	12·5	13·5	14·5	15·5	16·5	17·5	18·5	19·5	20·5	21·5
Frequency	6	87	94	118	73	41	24	19	16	3	8	4	3	4	0	2

Redness: Locality 3. Total frequency, 368.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
	6	7	8	9	10	11	12	13	14	15	16	17
Frequency	2	5	40	47	113	68	44	30	9	8	0	2

Blackness: Locality 1. Total frequency, 484.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
Frequency	1	1	1	1	4	5	3	8	25	34	49	76	97	106	55	15	3

Blackness: Locality 2. Total frequency, 502.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
	76·5	77·5	78·5	79·5	80·5	81·5	82·5	83·5	84·5	85·5	86·5	87·5	88·5	89·5	90·5	91·5	92·5
Frequency	2	1	2	4	6	7	7	11	16	15	33	65	91	99	73	61	9

Blackness: Locality 3. Total frequency, 368.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
	81	82	83	84	85	86	87	88	89	90	91	92	93	94
Frequency	2	5	6	16	28	27	41	60	73	65	28	14	2	1

Depth of Suture: Locality 1. Total frequency, 456.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51
Frequency	6	15	32	48	60	65	64	62	41	28	15	6	8	2	4

Depth of Suture: Locality 2. Total frequency, 498.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59
Frequency	6	42	76	102	102	78	46	15	14	9	4	2	1	0	0	0	0	1

Depth of Suture: Locality 3. Total frequency, 368.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
	8	13	18	23	28	33	38	43	48	53	58	63	68	73	78	83
Frequency	1	5	27	67	81	53	47	34	20	11	14	3	3	0	1	1

Depth of Suture: Locality 3a. Total frequency, 461.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
	18	23	28	33	38	43	48	53	58	63	68	73	78
Frequency	4	15	28	53	99	109	66	50	30	3	3	0	1

APPENDIX B.

Correlation Tables. The classes are defined by the class numbers of the variation statistics.

a. Correlation between length and number of whorls of shells of *N. obsoleta* from locality 3.

Length:
Mean = 5·0983.
Standard Deviation = 1·868.

Number of Whorls:
Mean = 5·8286.
Standard Deviation = 1·573.

Correlation = ·0640.

Total frequency = 356.

Length \ Number of Whorls	1	2	3	4	5	6	7	8	9	10
1	—	—	—	1	1	2	1	—	—	—
2	—	—	—	2	6	6	4	1	—	—
3	—	1	1	11	6	9	13	7	1	—
4	1	2	1	13	12	14	9	12	1	1
5	—	1	2	20	13	23	9	8	2	—
6	—	—	2	9	12	13	4	10	1	—
7	—	3	—	8	6	15	5	4	3	—
8	—	—	1	—	5	14	7	2	1	—
9	—	—	—	1	—	6	3	3	—	—
10	—	—	—	—	—	—	—	—	—	1

b. Correlation between length and number of whorls of shells of *N. trivittata* from locality 3.

Length:
Mean = 7·879.
Standard Deviation = 2·594.

Number of Whorls:
Mean = 9·5548.
Standard Deviation = 2·373.

Correlation = ·4178.

Total frequency = 438.

Length \ Number of Whorls	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	5	2	—	1	—	—	—	—	—	—	—	—
3	—	—	1	—	1	2	5	2	3	—	1	—	—	—	—	—
4	—	—	—	1	—	1	5	1	8	1	—	—	—	—	—	—
5	—	—	—	1	—	1	4	10	13	5	2	2	—	—	—	—
6	—	—	—	1	3	1	1	10	14	12	4	2	2	—	—	—
7	1	—	1	—	2	—	2	5	13	14	13	1	1	—	—	—
8	1	—	—	—	1	2	2	4	17	24	12	6	3	1	—	—
9	—	1	—	—	—	—	5	4	10	13	15	9	8	—	—	—
10	—	—	—	2	2	—	3	—	5	7	12	11	1	2	—	—
11	—	—	—	—	2	—	6	—	2	4	4	7	12	—	—	—
12	—	—	—	—	3	3	2	—	—	3	4	2	5	1	1	1
13	—	—	—	—	—	—	—	—	1	1	1	1	3	—	1	—
14	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—
15	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—

c. Correlation between length and number of whorls of shells of *N. obsoleta* from locality 1.

Length :
Mean = 7·856.
Standard Deviation = 2·2151.

Number of Whorls :
Mean = 4·505.
Standard Deviation = 2·1642.

Correlation = ·2860.
Total frequency = 493.

Length \ Number of Whorls	1	2	3	4	5	6	7	8	9	10
1	—	—	1	—	—	—	—	1	—	—
2	—	—	—	—	—	—	—	—	—	—
3	—	3	1	—	—	—	—	—	—	—
4	5	3	1	4	1	—	1	—	1	—
5	3	15	10	10	8	—	1	—	3	1
6	8	13	8	10	14	5	1	2	1	—
7	6	14	16	17	19	6	3	2	5	1
8	2	11	11	17	19	11	4	6	1	1
9	—	8	7	11	19	10	11	6	8	—
10	—	5	9	5	16	10	3	1	1	1
11	—	3	2	4	10	4	4	2	1	2
12	—	—	2	1	2	1	—	1	2	—
13	—	—	1	2	—	3	—	1	—	—
14	—	—	2	—	1	1	1	—	—	—
15	—	—	—	—	1	—	—	1	—	—

d. Correlation between length and ratio of diameter to length of shells of *N. obsoleta* from locality 3.

Length :
Mean = 5·136.
Standard Deviation = 1·902.

$\frac{\text{Diameter}}{\text{Length}}$:
Mean = 8·689.
Standard Deviation = 2·797.

Correlation = − ·2641.
Total frequency = 367.

Length \ Index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	—	—	—	—	—	—	—	—	2	1	2	—	—	—	—	—	—
2	—	—	—	—	1	1	1	2	6	1	3	1	3	—	—	—	—
3	1	2	1	—	2	2	5	6	6	5	4	9	5	2	—	1	—
4	—	—	—	2	3	7	6	7	7	9	13	8	2	—	—	2	2
5	—	3	1	5	1	8	6	13	16	6	7	6	3	2	—	—	1
6	—	2	—	—	2	2	10	7	13	10	3	2	2	1	1	—	—
7	—	—	—	1	3	10	8	5	8	6	4	—	—	1	—	—	—
8	—	—	—	1	3	10	6	4	3	1	1	2	—	—	—	—	—
9	—	—	—	1	2	1	2	4	3	—	—	—	—	—	—	—	—
10	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

e. Correlation between length and ratio of diameter to length of shells of *N. trivittata* from locality 3.

Length:
Mean = 7·9629.
Standard Deviation = 2·642.

$\frac{\text{Diameter}}{\text{Length}}$:
Mean = 6·3158.
Standard Deviation = 1·870.

Correlation = −·7193.
Total frequency = 456.

Length \ Index	1	2	3	4	5	6	7	8	9	10	11	12
1	—	—	—	—	—	—	—	—	1	—	1	—
2	—	—	—	—	—	—	—	—	—	1	5	2
3	—	—	—	—	—	—	—	—	6	5	4	—
4	—	—	—	—	1	—	1	7	3	3	2	—
5	—	—	—	—	2	2	11	15	8	1	—	—
6	—	—	—	—	2	12	10	15	8	2	—	—
7	—	—	—	—	11	18	15	6	4	—	—	—
8	—	—	—	7	21	25	17	3	1	—	—	—
9	—	—	—	10	24	26	10	1	—	—	—	—
10	—	—	3	10	15	12	5	—	1	—	—	—
11	—	—	1	10	17	10	4	1	—	—	—	—
12	2	1	2	7	9	7	—	—	—	—	—	—
13	—	—	1	5	1	—	1	—	—	—	—	—
14	—	—	1	—	—	—	—	—	—	—	—	—
15	—	—	1	—	—	—	—	—	—	—	—	—

f. Correlation between length and apical angle of shells of *N. trivittata* from locality 3.

Length:
Mean = 7·695.
Standard Deviation = 2·590.

Angle:
Mean = 5·722.
Standard Deviation = 2·047.

Correlation = −·6577.
Total frequency = 461.

Length \ Angle	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	2	—	—
2	—	—	—	—	—	—	—	2	1	2	3
3	—	—	—	—	1	—	1	2	6	4	1
4	—	—	—	—	—	2	5	3	6	1	—
5	—	—	—	—	1	11	12	12	3	—	—
6	—	—	—	3	2	12	16	5	9	2	1
7	—	1	2	4	11	16	13	5	2	1	—
8	1	—	3	17	19	17	14	4	1	—	—
9	—	4	9	19	15	17	7	—	—	—	—
10	1	4	6	12	14	5	2	2	—	—	—
11	—	4	5	9	10	10	5	1	—	—	—
12	2	5	4	9	3	3	2	—	—	—	—
13	—	1	5	—	1	1	—	—	—	—	—
14	1	—	—	—	—	—	—	—	—	—	—
15	—	—	1	—	—	—	—	—	—	—	—

g. Correlation between the ratio of diameter to length and the apical angle of shells of *N. obsoleta* from locality 1.

$\frac{\text{Diameter}}{\text{Length}}$:
Mean = 5·0973.
Standard Deviation = 1·759.

Angle:
Mean = 10·2292.
Standard Deviation = 2·946.

Correlation = ·4787.
Total frequency = 493.

Index \ Angle	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	—	—	1	—	—	—	1	2	1	—	—	1	—	—	—	—	—	—	—
2	1	—	—	—	5	1	2	—	3	2	4	1	—	—	—	—	—	—	—
3	—	2	1	2	—	5	7	6	9	10	7	3	3	—	1	—	—	—	—
4	—	1	1	5	4	9	20	16	12	13	15	17	1	2	—	1	—	—	—
5	—	—	—	—	2	6	6	11	16	11	16	16	12	7	—	—	—	—	—
6	—	—	—	—	1	1	2	10	9	16	13	18	9	9	3	2	1	—	—
7	—	—	—	—	—	1	—	3	3	7	6	9	8	8	5	2	2	—	—
8	—	—	—	—	—	—	—	—	2	1	1	6	5	5	3	3	1	—	—
9	—	—	—	—	—	—	—	—	1	1	1	1	2	2	2	—	—	—	1
10	—	—	—	—	—	—	—	—	—	1	1	1	1	1	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—

h. Correlation between the ratio of diameter to length and the apical angle of shells of *N. trivittata* from locality 3.

$\frac{\text{Diameter}}{\text{Length}}$:
Mean = 6·3158.
Standard Deviation = 1·870.

Angle:
Mean = 5·735.
Standard Deviation = 2·054.

Correlation = ·7941.
Total frequency = 456.

Index \ Angle	1	2	3	4	5	6	7	8	9	10	11
1	—	—	1	1	—	—	—	—	—	—	—
2	1	—	—	—	—	—	—	—	—	—	—
3	2	4	—	2	1	—	—	—	—	—	—
4	2	10	16	16	2	2	1	—	—	—	—
5	—	4	17	33	26	18	5	—	—	—	—
6	—	1	1	17	35	42	13	3	—	—	—
7	—	—	—	3	8	19	33	7	2	2	—
8	—	—	—	—	1	10	16	17	4	—	—
9	—	—	—	—	1	3	7	6	13	1	1
10	—	—	—	—	—	—	—	1	8	3	—
11	—	—	—	—	—	—	1	2	3	3	3
12	—	—	—	—	—	—	—	—	—	1	1

i. Correlation between number of whorls and apical angle of shells of *N. obsoleta* from locality 3.

Number of Whorls:
Mean = 5·832.
Standard Deviation = 1·615.

Angle:
Mean = 8·0784.
Standard Deviation = 2·859.

Correlation = ·0077.
Total frequency = 357.

Number of Whorls \ Angle	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
2	—	—	1	—	1	—	—	—	2	1	1	—	1	—	—	—
3	—	1	—	—	—	2	1	1	2	—	—	—	—	—	—	—
4	—	—	3	6	1	9	7	12	8	5	3	6	3	—	2	—
5	—	1	1	2	8	7	3	15	4	6	2	5	3	3	—	1
6	1	2	5	10	11	10	11	13	14	11	6	6	1	1	—	—
7	—	1	3	2	3	4	3	8	10	5	7	7	1	1	—	—
8	—	—	2	1	2	3	5	6	11	7	9	2	—	—	—	—
9	—	—	—	—	1	1	3	1	—	2	1	—	—	—	—	—
10	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—

ON THE AMBIGUITY OF MENDEL'S CATEGORIES.

By W. F. R. WELDON, F.R.S.

I. *Introductory.*

In the early part of this year I published an account of the results obtained by Mendel in his experiments with certain races of cross-bred peas, and of the subsequent attempts to show that results similar to his were obtainable in other cases*. I see no reason to modify the statements I then made.

Since my article appeared the Royal Society has published a Report by Mr Bateson and Miss E. R. Saunders†, in which a large number of experiments in cross breeding are described; Mendel's results are re-stated, and most of the observations recorded are held to be in substantial agreement with Mendel's laws. In this Report, and in a separate essay by Mr Bateson‡, an altogether new interpretation of Mendel's results is suggested, and an issue of considerable importance is raised.

Mendel says that the races of peas, used in his experiments, were observed during several generations, and were found to differ constantly in certain characters, for example in the colour of their cotyledons, which was green in some races, yellow in others. Now "green" and "yellow" are not quantitatively definite terms; each includes a considerable range of recognisably different colours, and every known race of peas produces seeds whose cotyledons vary in colour. All that can be inferred from Mendel's statement, therefore, is, that the range of variation in cotyledon colour, which some of his races exhibited, fell entirely within the range of colours called "green," while that of other races fell entirely within the range of colours called "yellow." Mendel accounts for the behaviour of hybrids between these races, as he describes them, by assuming that each hybrid bears gametes, male and female, of two kinds and in equal numbers, "und dass diese Keim- und Pollenzellen "ihrer inneren Beschaffenheit nach den einzelnen Formen entsprechen§." The

* See *Biometrika*, Vol. I. p. 228.

† *Reports to the Evolution Committee: I. Experiments undertaken by W. Bateson, F.R.S., and Miss E. R. Saunders*, Royal Society, 1902.

‡ *Mendel's Principles of Heredity, a Defence:* by W. Bateson. Cambridge University Press, 1902.

§ *Verhandl. d. naturforsch. Vereins Brunn*, Bd. IV. p. 29.

issue raised by Mr Bateson and Miss Saunders concerns the meaning which it is necessary, in the light of known facts, to give to this expression.

The result, described by Mendel, may be illustrated by reference to the following diagram:

Let AD be a scale of colour,—a map of a spectrum, or some similar thing,—of which the range AB includes the various shades of green, the range CD those of yellow. A race of peas has cotyledons whose colour varies, in individual cases, within the range AB; a second race has cotyledons which vary within the range CD. If these races be crossed, the hybrid cotyledons are said to fall within the colour-range CD; and the plants to which they give rise will, on Mendel's hypothesis, produce gametes of two kinds, in equal numbers; those of the first kind, if paired, give rise to plants whose cotyledon colour (and that of their descendants) lies within the range AB; those of the second kind, if paired, give rise to descendants whose cotyledon colour lies within the range CD.

That is to say, the descendants of such a cross fall into two groups, each referable, so far as cotyledon colour is concerned, to the same category as one of the ancestors used in the original cross. We are quite unjustified, from the data given by Mendel or by any of those who follow him, in saying more than this. Thus the green-seeded descendants of such a cross are described as "green": they are not described, by Mendel or by any of his followers, as being of the same shade of green as the particular member of a variable race used in the original cross. The information given is therefore compatible with either of several theories of the constitution of those gametes in the hybrid from which "recessive" individuals arise. For anything we are told to the contrary, the character transmitted by such gametes may be a blend of all the kinds of green exhibited by their green-seeded ancestors in various proportions; or different gametes may revert directly to the colour of different individual ancestors; or finally all the recessive gametes may transmit the characters of the green-seeded ancestor which took part in the original cross, and of no others. The first two hypotheses involve the belief that the composition of gametes of either kind, whether "dominant" or "recessive," is affected by that of a whole series of ancestors. Such belief seemed (and still seems) to me a necessary consequence of the facts. Mr Bateson and Miss Saunders have however adopted the view that "the pure dominant and the pure recessive members "of each generation are not merely like, but identical with the pure parents" (*l. c.* p. 12); and Mr Bateson denounces in no measured terms my attempt to regard the inconsistent results obtained by those who have repeated Mendel's experiments as due to differences in the ancestry of the races used.

Mr Bateson fully admits the variability of all races of peas; and he admits that the range of colour-variation in many races is so great that they include colours of both the "green" and the "yellow" category. Nevertheless, he believes that many of these races behave, when crossed, like those described by Mendel. This

admitted variability of the races makes it possible for Mr Bateson to test his view experimentally; for if a "recessive" plant, resulting from a cross, is "not merely like but identical with" one of the pure-bred plants used in the cross, then the peas on the recessive plant must exhibit exactly the same series of colour-varieties in their cotyledons as those exhibited by the pure-bred plant which it resembles. Before Mr Bateson can justify the view he has put forward, he must therefore not only find two races which obey Mendel's laws when crossed, which he says he has done, but he must determine

(1) The variability of cotyledon colour in each race;

(2) The mean cotyledon colour and its variability in each plant used in crossing;

(3) The mean cotyledon colour and its variability in each "recessive" and in each "dominant" plant descended from the cross.

Until the result of such determinations is known, it is impossible to distinguish between the resemblance of a series of cross-bred plants to one of the ancestral races, and their resemblance to an individual plant of that race; so that Mr Bateson's contention cannot be supported by evidence.

The confusion between resemblance to a race and resemblance to an individual involved in Mr Bateson's treatment of Mendel's work is one of the many unfortunate results which follow when Mendel's system of dividing a set of variable characters into two categories, and of using these categories as statistical units, is carried too far. Unless the range of characters actually included in each category be constantly borne in mind, the degree of resemblance between two individuals, implied by placing them in the same category, cannot be estimated; and when, as constantly happens, the range of variation in one of the alternative categories differs widely from that included in the other, the Mendelian system becomes absolutely misleading without some explanation (nearly always withheld) of the real limits implied by the terms used. Thus if two plants are said to be glabrous, we know that they are absolutely similar in so far that they possess no hairs; if they are said to be hairy, we know that they both possess hairs, but one may, for anything we are told to the contrary, have ten times as many hairs, per unit of surface-area, as the other. Again the tendency to apply two categories, found suitable for a particular race, to other races of the same or allied species leads to very harmful results; for example the classification of peas into those with green and those with yellow cotyledons leads to a quite erroneous conception of the distribution of cotyledon colour in most existing races of peas, although it may have expressed the facts observed by Mendel in the races which he used.

The Report of Mr Bateson and Miss Saunders contains many statements which would, I think, never have been made if the authors had not been misled by the use of Mendelian categories. It is impossible to realise the meaning of evidence, brought forward to prove that particular hybrids behave in the manner described by Mendel, unless the meaning of the categories employed in each special case is

clearly understood. I therefore propose, as opportunity offers, to describe the variability of such races as I can obtain, among those which are said to obey Mendel's laws, and to consider how far the statements made concerning them are affected by the employment of Mendel's very imperfect system of units.

II. *Lychnis diurna* and *Lychnis vespertina.*

In the Report by Mr Bateson and Miss Saunders the results of crossing normal *L. vespertina* and *L. diurna* with white and red-flowered glabrous varieties are described; it is said that the phenomena "follow Mendel's law with considerable accuracy, and no exceptions that do not appear to be merely fortuitous were discovered" (*l.c.* p. 15). Apart from the great uncertainty involved in the use of "hairy" as a definite category alternative to "glabrous" (which will be dealt with later) these experiments are of interest, because a parallel series of crossings had already been carried out by Professor de Vries, who used the same glabrous varieties as those used, at least in some cases, by our authors. They express themselves as "specially indebted to Professor de Vries" for them; and they add in a footnote, "The discovery by de Vries of a wild specimen of *L. vespertina* var. "*glabra*, and the artificial production from it of a smooth red-flowered form are "described in his 'Erfelijke Monstrositeiten,' p. 10...." Now I have not been able to find a work by Professor de Vries, under this title, which contains a page 10; but in the *Botanisch Jaarboek*, Jaargang IX. 1897, pp. 62—93, there is a paper by him entitled "Erfelijke Monstrositeiten in den ruishandel der botanischen tuinen." Under the heading *Lychnis vespertina glabra* I find the following statement (p. 71): "In August 1888 I collected seed of *Lychnis vespertina* in a wood "near Hilversum. Among the plants produced in my experimental garden in "the following year were some completely glabrous examples. I isolated these, "and in the course of some generations the glabrous race has been rendered "stable (geheel standvastig) as a result of continued selection (door voortdurende "selectie). It seems never, or only very rarely, to produce atavistic individuals.

"Traces of hairs are still to be found here and there, especially upon the young "plants. This point is worth closer investigation."

This race of "glabrous" plants was therefore established by a process of selection, and seven or eight years after it was first observed it still produced traces of hairs. There is no evidence in the account given to show whether the seed came originally from a glabrous wild plant, as suggested by Mr Bateson and Miss Saunders, or not; but they may have fuller information from Professor de Vries. In any case the first glabrous plants observed did not behave as "mutations," but as we should expect extreme variations from the hairy type to behave; so that several generations of selection were required in order to fix a stable race with their characters. Although its establishment by continued selection suggests that this glabrous race had not at first the properties of a "recessive" race in Mendel's

sense, yet it is said to have behaved as a strictly recessive form when crossed with hairy races later on. In 1892 it was crossed with normal forms of *Lychnis diurna*, and the resulting generations were held to obey Mendel's laws. The hybrids of the first generation were all hairy; and the offspring obtained by pairing these were in part hairy and in part glabrous. From the glabrous hybrids a stable glabrous variety is said to have been raised, and there is no record that these glabrous "recessive" forms ever give rise to "hairy" plants. The gametes of the glabrous *Lychnis vespertina*, as fixed by selection, are therefore said to behave in this cross as if they were what writers on Mendel's work call "pure." Mr Bateson rejects the view that the characters of cross-bred individuals, derived in part from such "pure" parents, can be regarded as depending upon the characters of the ancestors from which the "pure" parents are descended; he declares that they depend entirely upon the characters of the "pure" individuals used in making the cross. It is therefore to be regretted that he has abstained from discussing an experiment in which Professor de Vries crossed this "pure" glabrous *Lychnis vespertina* with *Silene noctiflora*, and obtained hybrid offspring which were indeed hairy, but their hairs were of the type proper to *Lychnis vespertina*, and not of the type of *Silene noctiflora*. Surely we have here a clear proof that the "dominant" character, hairiness, may on the application of a suitable stimulus be manifested by the fertilised germ-cells of what is said to be a purely recessive plant; so that the theory of pure parental gametes, on which Mr Bateson lays such stress, is shown to be inadequate for this case, and a theory of inheritance, with reversion to particular ancestors, is indicated as likely to express the facts of "Mendelian" cases also.

As has already been said, Professor de Vries crossed the glabrous *L. vespertina* here described and normal hairy *L. diurna*, always apparently using *L. diurna* ♀ × *L. vespertina* ♂; the first generation of hybrids contained only hairy individuals. The second generation is not very fully described; in the *Erfelijke Monstrositeiten* (p. 72) we are told that about $\frac{2}{3}$ of the individuals were hairy, and $\frac{1}{3}$ glabrous; in the *Comptes Rendus*, March 1900, we are told that 28 per cent. were glabrous; and this statement is repeated in subsequent accounts.

Although the actual numbers of individuals are not given, so that the probable errors of these results cannot be calculated, it is clear from the adoption of the round numbers $\frac{1}{3}$ and $\frac{2}{3}$ that the impression produced was not that of $\frac{1}{4}$ recessive and $\frac{3}{4}$ dominant or dominant-hybrid individuals, as it should have been on Mendel's hypothesis. Several hundred of individuals are said to have been observed. The odds against a deviation so largely exceeding Mendel's result with 500 individuals would be about 17 to 1. The glabrous variety produced in these experiments appears to have furnished some, at least, of the glabrous, red-flowered forms used by Mr Bateson and Miss Saunders, and in their hands it is said to have given results in good accord with Mendel's law; they pass over the deviations from Mendel's law, observed during its earlier history, without notice.

So much for what is known about the previous history of the races used by Mr Bateson and Miss Saunders. It is sufficient to show that even the use of Mendel's categories is not enough to bring the phenomena described into anything like *exact* accord with Mendel's laws. Before going further, we will examine a little more closely the conditions included under the two categories "glabrous" and "hairy."

By *glabrous* Professor de Vries understands (at least when speaking of *L. vespertina*) the absence of perfect hairs from all parts of the plant, although he calls a variety "glabrous" when especially in the young state it has "traces of hairs" here and there. Mr Bateson and Miss Saunders leave their meaning a little doubtful. On p. 16 they say "in the glabrous varieties no hairs were observed on any part of the plants at any time"; but since when speaking of cross-bred forms they always refer to hairiness as a character of *leaf-surface*, it is not quite clear to me whether a cross-bred plant with glabrous leaves and a hairy stem was ever observed by them, and if so, whether they would classify it as glabrous or as hairy. The only glabrous plant I have myself found wild (*L. vespertina*) had no hairs on any part of its surface; and I am not aware that plants with hairy stems or calices and glabrous leaves have been described. For this reason the repeated statement that hairiness is a character of leaf-surface is difficult to understand.

The category "*glabrous*," whether merely applied to leaf-surface or to the whole plant, certainly includes only a small range of conditions. It is far otherwise with the category "*hairy*." In accordance with the differences in habit of the two species, the hairs of *L. diurna* and of *L. vespertina* differ considerably in their distribution, even when leaf-surface only is considered; and while the average condition of the hairs in the two "species" is different, each "species" varies from race to race, and from individual to individual.

The hairs are multicellular, and may vary in length, on the same leaf, from about 0·2 mm. to about 1·9 mm.; the cell sap may be red, or blue, or nearly colourless; some of the hairs on a leaf may be glandular, the percentage of such hairs varying greatly. Further, when a race of *L. diurna* and one of *L. vespertina*, both being hairy, are crossed, the hybrids are said by Gagnepain to be intermediate between the parent races in the condition of their hairs (*Bull. Soc. Bot. de France*, T. XLIV. p. 445).

In order to avoid the effects of differentiation among the leaves, it is necessary to compare corresponding leaves of the different plants examined. To illustrate the variation in the frequency with which hairs occur in unit area of leaf-surface Table I. has been compiled. It shows the number of hairs per square centimetre of epidermis on the lower side of a leaf from the node below that which bears the terminal flower. The observations were made (*a*) upon a race found in a little copse on the Berkshire (right) bank of the Thames, about four miles by road below Oxford, and close to Sandford Lock; (*b*) upon a race found on the slopes of Cooper's Hill, Surrey, also on the right bank of the Thames, but more than 35 miles distant

from Sandford in a straight line. The second race was collected, and in large part examined by Miss C. B. Sanders, whom I gladly take this opportunity of thanking for the time and labour she was kind enough to give. The epidermis was removed from the leaf, and mounted in water, with its hairy side uppermost, the hairs being then counted under a low power of the microscope; in this way error due to the accidental breaking of hairs was avoided, because the stump of a broken hair was easily recognised.

TABLE I.

Number of Hairs per square centimetre of Lower Leaf in Lychnis diurna ♀. *The area chosen in all cases so as to exclude the great veins.*

Number of Hairs	Frequency		Number of Hairs	Frequency	
	Sandford	Cooper's Hill		Sandford	Cooper's Hill
0— 24	0	1	350—374	6	—
25— 49	1	10	375—399	1	—
50— 74	2	14	400—424	6	—
75— 99	5	19	425— 449	1	—
100—124	5	17	450—474	1	—
125—149	7	11	475—499	0	—
150—174	17	5	500—524	1	—
175—199	9	8	525—549	0	—
200—224	7	5	550—574	1	—
225—249	9	3	575—599	0	—
250—274	7	2	600—624	0	—
275—299	3	2	625—649	0	—
300—324	7	3	650—674	1	—
325—349	4	—	675—699	1	—
			Totals	102	100

The figures given do not convey an adequate impression of the range of variability throughout the species. Even in localities close to those inhabited by the races described a greater range of variation has been observed; such as they are, however, the two races examined are enough to show how wide is the range, covered by the category "hairy," in *L. diurna* alone. In a single race of *L. vespertina* from Shotover Hill, near Oxford, it is even greater; 112 plants of this race included one glabrous individual, and one with 1106 hairs per square centimetre on the standard leaf. The distribution of hairs in this race is given in Table II, which is again an inadequate picture of the range throughout the species, even in the neighbourhood of Oxford. Odd plants gathered by the wayside have given over 1300 hairs per square centimetre.

TABLE II.

Number of Hairs per square centimetre on Lower Leaf Surface in Lychnis vespertina ♀ from Shotover.

Number of Hairs	Frequency	Number of Hairs	Frequency
0— 24	1	500—524	5
25— 49	0	525—549	4
50— 74	0	550—574	0
75— 99	1	575—599	3
100—124	2	600—624	3
125—149	4	625—649	2
150—174	2	650—674	1
175—199	6	675—699	2
200—224	5	700—724	2
225—249	4	725—749	0
250—274	7	750—774	0
275—299	5	775—799	1
300—324	5	800—824	0
325—349	3	825—849	0
350—374	9	850—874	3
375—399	7	875—899	0
400—424	4	900—924	1
425—449	9	925—949	0
450—474	5	950—974	1
475—499	4	975—999	0
		* *	*
		1100—1124	1
		Total	112

Taking a group of 25 hairs as the unit of measurement, and calling the group containing from 0 to 24 hairs the first, we have, for the constants of the three distributions the following values*:

TABLE III.

Constants of the Frequency. Distribution of Hairiness.

	L. diurna ♀		*L. vespertina* ♀
	Sandford	Cooper's Hill	Shotover Hill
Mean	10·2255	5·4900	16·7143
Mode	7·0284	—	14·4399
σ	4·8863	2·7725	7·9583
μ_3	132·0653	—	441·4132
μ_4	2658·4536	—	15675·9832
β_1	1·281485	—	0·766935
β_2	4·663582	—	3·907930
a_1	4·4337	—	13·6382
a_2	92·7746	—	158·3939
m_1	1·1689	—	2·5372
m_2	24·4607	—	33·6951

* Notation is that used by Pearson: *Phil. Trans.*, Vol. 186, A, p. 367 et seq.

The probable error of all these values is of course considerable, but they fit the observations as well as could perhaps be expected. Thus the lower limit of hairiness in the race of *L. diurna* from Sandford is $7{\cdot}0284 - 4{\cdot}4337 = 2{\cdot}5947$ units, or allowing for the probable error of the start of the range, which is about ·63 units, $52 \pm {\cdot}16$ hairs, the lowest observed number of hairs being 47. I feel bound to call attention to this feature of the distribution, which is some sign that in *L. diurna* the range of variation, in a normal hairy race, does not involve an occasional production of glabrous individuals; it is interesting in this connection to notice (1) that Mr Bateson and Miss Saunders carefully refrain from speaking of a glabrous variety of *L. diurna*, and (2) that the red-flowered hybrid, which Professor de Vries calls *Lychnis diurna glabra*, results from the cross between glabrous *L. vespertina* and hairy *L. diurna* already alluded to. The lower limit to the distribution in *L. vespertina* is $14{\cdot}4399 - 13{\cdot}6382 = 0{\cdot}8017$ units, or 8 hairs per centimetre, which is as good an approximation to a lower limit at no hairs as we need expect from the limited number of observations. It must be remembered that the first unit of the grouping adopted has a centre at 13 hairs, while the only individual in the group was glabrous.

The Cooper's Hill material seems heterogeneous, and I have not thought it worth while to resolve this small number of individuals into components.

TABLE IV.

Number of Hairs with Glandular Extremities per Hundred Hairs observed.

Gland Hairs per cent.		*L. diurna* ♀		*L. vespertina* ♀
		Sandford	Cooper's Hill	Shotover Hill
0—4·9	0	89	100*	43
	Less than 1	3	—	12
	1—1·9	3	—	2
	2—2·9	1	—	2
	3—3·9	1	—	6
	4—4·9	2	—	3
5·0— 9·9		1	—	16
10·0—14·9		1	—	6
15·0—19·9		1	—	5
20·0—24·9		0	—	6
25·0—29·9		0	—	1
30·0—34·9		0	—	0
35·0—39·9		0	—	0
40·0—44·9		0	—	1
Totals		102	100	103

* So my notes say. If we make the utmost allowance for possible breakage of the tips of gland-hairs, so that they cannot be recognised, it is I think certain that no female plant from Cooper's Hill had one per cent. of such hairs even near the base of the leaf.

The foregoing facts show the range of variation in the number of hairs per unit area on the leaves of a few individuals. It is difficult to estimate the percentage frequency of glandular hairs, because they are more frequent near the base of the leaf than near its apex; I have attempted an estimate of their percentage frequency in the basal part of the leaf, and the result is given in Table IV.

The few data here brought together are sufficient to show the way in which the adoption of such a category as "hairy" conceals the facts of variation within the races discussed. In the light of such facts the statements made by Mr Bateson and Miss Saunders are seen to be utterly inadequate, either as a description of their own experiments, or as a demonstration of Mendel's or of any other laws.

When hairy and glabrous plants were crossed we are told that "among the whole number of plants raised, not a single intermediate was observed" (*l.c.* p. 15), but we are not told what "an intermediate" is. The authors must know, or they could not certify its absence; it would have been well if they had thought fit to define an "intermediate" in their Report, for the definition is a vital part of their argument.

The preceding tables show that "hairiness" is not an absolute, invariable quality, but that it is manifested in various degrees. In the few individuals examined it is possible to pass by a series of small steps from the glabrous condition through individuals with various numbers of hairs per square centimetre of leaf-surface, up to a condition of very great hairiness. It is perfectly legitimate to regard those individuals with a small number of hairs per unit area as intermediate between those with a larger number and those with a smaller number or with none. Thus a plant with only 80 hairs per unit area of leaf-surface may be called intermediate between a plant with 200 hairs per unit area and a plant with none.

Other conditions are conceivable, intermediate between that of a plant with a given number of hairs per unit area, the hairs being of known length, and that of a glabrous plant. Thus the transition might take place by a reduction in the length of the hairs through various steps to zero, without reduction in their number; in which case the plant with the shorter hairs would be intermediate between the plant with longer hairs and the glabrous individual; or again the transition might conceivably be effected by the appearance of glabrous patches, so that the intermediate individuals assumed a mosaic character. On the whole, however, the density of the hairs per unit area seems the best measure of "hairiness" for our present purpose.

Whatever we may choose as a measure of hairiness, Mr Bateson and Miss Saunders give no evidence by which we can judge the result of their work. Thus we are told that "hairiness" is dominant, because "of the thousand cross-breds "raised from various unions between hairy and glabrous strains, all, without excep- "tion, were hairy" (*l.c.* p. 19); but unless we know how hairy they were, we cannot judge what is the value of the statement that hairiness is dominant. Thus a plant

with 40 or 50 hairs per square centimetre is obviously and unquestionably hairy; it is just as clearly intermediate between a plant with 1100 hairs per centimetre and a glabrous plant. Without some proof that the mean number of hairs per unit area was not less in the offspring of a cross between a glabrous and a hairy form than it was in the hairy parent, the result described cannot be distinguished from that most probable on any theory of "blended" inheritance.

It is most unfortunate that the degree of hairiness of each individual was not recorded, because the great variability of the parental races would have given an excellent opportunity of demonstrating the truth of the statement that pure dominants "are not merely like, but identical with the pure parents." We might at least have hoped to learn whether a *Lychnis diurna* with 40 hairs per centimetre, and a *L. vespertina* with 1100 or 1200, produce plants with different degrees of hairiness when crossed with the same glabrous variety, or whether the presence or absence of gland hairs on the leaves of the hairy parent has any effect upon this "character of leaf surface" in the offspring!

Unless all the hairy parents used, of either race, were equally hairy, the first cross-bred generation must either have contained individuals of different degrees of hairiness, or the identity between pure dominants and their parents disappears. On the other hand, if the cross-breds and their parents differed in this character we have no way of distinguishing "dominance" from the result of "blended" inheritance, until we abandon the Mendelian categories, and adopt a rational way of measuring hairiness. The absence of glabrous cross-breds will not help, until we know the variability of the hairy race, and the character of the hairy parent; for the chance of obtaining a glabrous individual among 1000 plants of any ordinary race of either species is admittedly small, and the variability of a few families, in each of which one parent is of fixed character, will on any hypothesis be less than the variability of the race, and the chance that such a series of families would contain a glabrous individual, on any theory of inheritance, cannot be estimated without information which is at present not available.

The difficulties which arise from imperfect description when we consider the question of dominance in the first cross-bred generation are equally formidable in the case of later generations. The category hairy is so wide that it is impossible to judge how the individuals included in it resemble or differ from their parents. The total results of the various crosses recorded are in better agreement with Mendel's statements than is usual, but the published data do not afford material for discussing the question how far any particular theory of inheritance can be successfully applied to them.

It is deeply to be regretted that so many interesting experiments, involving so much time and labour, should be recorded in a form which makes it impossible to understand the results actually obtained, and so gives rise to misconceptions both in the minds of the recorders and in others. Such justification as there may prove to be for classifying the form of inheritance exhibited by the hairs of *Lychnis* with

that described by Mendel may be just as easily seen when the variations observed both in parents and in offspring are clearly described; when this has been done, and not before, we shall be able to consider seriously what the issues raised by Mendel's statements really are. In the mean time the accumulation of records, in which results are massed together in ill-defined categories of variable and uncertain extent, can only result in harm.

The confusion introduced by the use of such categories into the record of experiments on the spiny-fruited and smooth-fruited races of *Datura*, performed by Mr Bateson and Miss Saunders, is even greater than that we have examined; but I refrain from discussing it or other cases in detail, until I can illustrate the categories by reference to a full series of specimens.

COOPERATIVE INVESTIGATIONS ON PLANTS.

I. ON INHERITANCE IN THE SHIRLEY POPPY*.

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(1) *Material.* In 1899 at Hampden Farm House on the Chilterns we had at our disposal for the study of homotyposis a considerable strip of garden covered with Shirley poppies. These were extremely fine plants, individuals at the end of the season having often 20 to 30 and occasionally 60 to 80 fruits. An investigation of the stigmatic bands of these capsules showed marked individuality in plants grown under apparently very like conditions of soil, air, rain and individual crowding. Of course plants on the borders of the plot had rather more opportunity for side development and put forth—on the whole late in the season—poor flowers on low shoots. The capsules of these low flowers had rarely any quantity of fertile seed and often none at all. The differentiation of high and low flowers in size of capsule was not accompanied by an equally marked differentiation in the number of stigmatic bands. Owing to this and the relative fewness of outlying individuals the whole plot was dealt with as producing a fairly homogeneous mass of plants growing under like conditions. For a first approximation capsule differentiation was neglected. This rule has been adopted in later crops;

* Drafted by K. Pearson.

the refinement of separating a small percentage of outlying individuals being more apparently than really advantageous. For, given the same soil and climate, there must, if any considerable number of plants be dealt with, be subtle individual advantages or disadvantages to individual plants in the crop due to slight differences in soil and situation, to thinning, tending or insect attack. Similar advantages or disadvantages, however, occur in the case of individuals, who are offspring of the same parents in men, horses or dogs, and do not therefore destroy all possibility of comparative study.

In 1900 F. W. Oliver also grew a crop of Shirley Poppies at Chelsea. These showed a very marked difference in the mean number of stigmatic bands per capsule as well as in the number of capsules per plant. We have then to recognise the factors: (i) considerable individuality as expressed by the homotypic correlation in plants grown under like environment, (ii) considerable difference in mean result for two crops grown in different environments, i.e. with differences of soil, tending, climate, atmosphere, etc.

The Shirley Poppy seemed excellent material for testing the laws of inheritance in plants, partly because of the healthy strong crops which can be raised under all sorts of conditions with small amount of attention; partly, because of the ease with which the stigmatic bands can be counted. In some of the series to be considered below colour has also been recorded, but the difficulties attaching to a satisfactory study of individual flowers have permitted at present only of limited consideration of this point. Further experiments in this matter are in progress.

In 1900 K. Pearson collected (i) all the seed from every capsule of 24* individuals out of his 176 Hampden plants; (ii) the seed from the capsules of a great variety of poppies sorted into groups of capsules having 8 to 18 bands; (iii) a mass of seed from all capsules and plants without discrimination. We had thus three lots of seed to be known as: (α) Individual Plant Seed: (β) Individual Band Seed and (γ) General Seed. In later years an additional type: (δ) Individual Capsule Seed, was dealt with.

From these lots of seed the following crops were grown in 1900:

(A) *Highgate Crop.* By the courtesy of the Misses Sharpe this was sown in a piece of meadow broken up for the first time for garden purposes. The soil was very poor and purposely not enriched in any way, the crop had an extremely hard struggle for existence at all, only a small percentage of the seed reaching maturity; the crop needed no thinning and was left to itself. There were only two or three capsules per plant, and the sizes of these as well as of the plants themselves were inferior even to F. W. Oliver's Chelsea crop of the preceding year. Only seed of type (α) was dealt with, (γ) failing entirely. In well tended and watered beds

* It would doubtless have been better to start with more than 24 individual plants, but besides selecting plants with a sufficient number of capsules, we had by the conditions of the homotypic experiments to wait till the end of the flowering season, and then many of the plants had scattered their seed.

of the same garden non-experimental (γ) seed gave strong healthy plants of plenty of blossom. Counting and reduction by K. Pearson.

(B) *Oxford Crop.* W. F. R. Weldon had seed types (α), (β) and (γ). The plants were raised in pots, and besides the stigmata a considerable number of characters on the individual plants was observed. The plants were raised in pots for ease in measurement and handling of individuals, but the need for constant watering introduced other difficulties to be referred to later. Counting and reduction by W. F. R. Weldon and in the case of the reduction of "first flower" characters by Alice Lee.

(C) *Chelsea Crop II.* F. W. Oliver had seed types (α) and (β). This crop was hopelessly ruined by a storm. The plants flowered well but the young fruits got beaten down and rotted off. Only 355 capsules were available for counting, one or two to a plant, and the frequency distribution of these seems the only data worth dealing with.

(D) *Crockham Crop.* A. G. Tansley had seed types (β) and (γ), and grew a very healthy crop. The counting and reducing are due to Marie A. Lewenz.

(E) *Bookham Crop.* G. U. Yule grew a crop of (α) seed. The plants were fairly healthy, but the crop was not fully harvested and counted. Counting is due to G. U. Yule and reduction to Alice Lee.

(F) *Enfield Crop I.* W. R. Macdonell grew a good crop from both (α) and (β) and also a control series of (γ). From his seed were taken supplies for the following crops in 1901:

(G) *Enfield Crop II.* This was grown from seed of type (δ). From the capsules of the Enfield crop of 1900, 57 were taken from different plants with bands varying from 9 to 20. 13 of the series failed, but the remaining 44 gave 239 plants with 566 capsules—a poor crop compared with that of 1900. Mrs W. R. Macdonell counted both Enfield crops, and the reductions are by W. R. Macdonell himself.

(H) *Kidderminster Crop.* This was grown by John Notcutt from seed of type (α) for 100 Enfield plants. The crop was a magnificent one, giving 1618 individual plants bearing more than 19,000 capsules. The work of counting the bands on these capsules is due to Margaret Notcutt, Marie A. Lewenz, Edna Lea-Smith and Norman Blanchard. The whole of the labour of reduction is due to W. Palin Elderton, one of whose tables involves upwards of 3,800,000 entries.

It will be seen that the material used for the purpose of the present paper is drawn from fairly diverse districts: London (2 crops), Oxford, Hertfordshire (2 crops), Surrey (2 crops) and Worcestershire. Further, it enables us to appreciate the magnitude of parental, grandparental and fraternal relations in plants, and the influence of environment, in the case of a single character in one species. There are many difficulties and obscurities, which require special investigation, and it is hoped that other experiments now in progress will throw light on some of these matters.

(2) *Complexity of Heredity in Plants.* With plants which are not either artificially crossed, or artificially restricted from crossing, the conception of the fraternal relation becomes very complex. If any artificial crossing or restriction in crossing be adopted, we can confine our attention to one special type of brother-plants, but not only is it very difficult to carry out such crossing or restriction in the large populations which alone seem to give reliable results in heredity, but when carried out we have reached a condition of things very different from what happens in wild life, where except for absolutely self-fertilising plants the diversity in brother-plants is itself a factor of the variation which evolution finds to draw upon. If we examine the case of man we find only three types of fraternal relationship, i.e. whole siblings or half-siblings on mother's or father's side. But in plants great complexity is introduced by the multiplicity of ovaries and pollen sacks, and by the possibility of self- or cross-fertilisation. If we put on one side (i) the division of the ovary into cells, and (ii) the division of the anther into lobes, we have still (iii) the multiplicity of anthers on the same flower and (iv) the multiplicity of flowers on the same plant to deal with. Are we to consider the plant as an individual because it proceeds from a single seed, and to suppose the rudiments of future seed in all cells of the same ovary and in all ovaries of the same plant of equal valency? And again is all pollen whether from anthers of the same flower, or from anthers of different flowers of the same plant, equivalent from the standpoint of parentage? Or, is the plant to be looked upon as a colony of individuals, and the flower to be the unit of parentage? We may even go beyond this and consider the individual cell of the ovary or the individual anther as our unit. It is clear that only very wide-reaching direct experiments on artificial fertilisation would enable us to distinguish whether one or other of the many types of plant parentage give offspring more or less alike. Still some classification of sibship in plant life seems desirable if we are to compare our poppy results with those for insects and animals.

We confine our attention for the time to the flower as the unit of parentage, and denote by a single letter an individual plant; small letters a, a', a''... denote ovaries of different flowers of this plant, large letters A, A', A''... the groups of anthers attached to the corresponding flowers or ovaries a, a', a''.... A pair of plants growing from seeds taken from the cells of the same ovary will be termed *co-ovarial*; if they come from two ovaries of the same plant *bi-ovarial*, if from ovaries on different plants *dis-ovarial*. If the pair of plants come from the pollen of anthers on the same flowers, they are *syn-anthic*, if from anthers of different flowers of the same plant *di-anthic*, and if from pollen of different plants *dys-anthic*. When fertilisation is from the anthers attached to a given ovary, we shall term the resulting plant *homotropic*; when fertilisation is from the anthers of another flower of the same plant, the result is *endotropic*, and when the pollen is from a different plant *exotropic*; the crossings for the same are *homogamic**, *endogamic* and *exogamic*. Given a pair of plants sprung from the ovaries of the same plant or from pollen of

* Not to be confused with the original use of this word for plants which mature the organs of both sexes at the same time.

the same plant, these may be termed siblings or half-siblings, and may be classified by compounding the above terms, thus instead of speaking of a homotropic-homotropic pair we can without confusion speak simply of a homotropic pair, and instead of an exotropic-homotropic or exotropic-endotropic pair, simply of an *exhomotropic* or an *exendotropic* pair. Further an interesting series of subcases arises when there is a mutual interchange of pollen between two flowers or plants. If the interchange be such that the ovary of one flower (a) receives pollen from the anther of a flower on the same or on a second plant (A' or B), while the ovary of this latter flower (a' or b) receives pollen from the anthers (A) associated with the first ovary, we shall speak of this relation as *hypermetatropic*. If the first ovary (a) receives from an anther (A' or B) associated with the second ovary (a' or b), but the second ovary from the anthers of the first plant not associated with the first ovary (A''), then the relationship is *mesometatropic*. Finally if the interchange be between two plants the pollen of one going to the other, but not the pollen from anthers associated with the ovaries fertilised, then the relationship is *prometatropic*. If the interchange is only half completed, then we have the three corresponding types of *hemimetatropy*.

We are now able to classify the various forms in plants corresponding to the fraternal relationship.

Full and Half-Sibship in Plants.

	Terminology	Symbol
I.	Co-ovarial synanthic homotropy	$aA \,.\, aA$
	„ dianthic endhomotropy	$aA \,.\, aA'$
	„ synanthic endotropy	$aA' \,.\, aA'$
	„ dianthic endotropy	$aA' \,.\, aA''$
	„ dysanthic exhomotropy	$aA \,.\, aB$
	„ dysanthic exendotropy	$aA' \,.\, aB$
	„ synanthic exotropy	$aB \,.\, aB$
	„ dianthic exotropy	$aB \,.\, aB'$
	„ dysanthic exotropy	$aB \,.\, aC$
II.	Bi-ovarial dianthic homotropy	$aA \,.\, a'A'$
	„ synanthic endhomotropy	$aA \,.\, a'A$
	„ dianthic endhomotropy	$aA \,.\, a'A''$
	„ dysanthic exhomotropy	$aA \,.\, a'B$
	„ dianthic hypermetatropy	$aA' \,.\, a'A$
	„ dianthic hemi-hypermetatropy	$aA' \,.\, a'A''$
	„ dysanthic hemi-hypermetatropy	$aA' \,.\, a'B$
	„ synanthic endotropy	$aA'' \,.\, a'A''$
	„ dianthic endotropy	$aA'' \,.\, a'A'''$
	„ dysanthic exendotropy	$aA'' \,.\, a'B$
	„ synanthic exotropy	$aB \,.\, a'B$
	„ dianthic exotropy	$aB \,.\, a'B'$
	„ dysanthic exotropy	$aB \,.\, a'C$

III.	Dis-ovarial synanthic exhomotropy *	$aA \,.\, bA$
	„ dianthic exhomotropy	$aA \,.\, bA'$
	„ synanthic exendotropy	$aA' \,.\, bA'$
	„ dianthic exendotropy	$aA' \,.\, bA$
	„ dysanthic hypermetatropy	$aB \,.\, bA$
	„ dysanthic mesometatropy	$aB \,.\, bA'$
	„ dysanthic hemi-hypermetatropy	$aB \,.\, bC$
	„ dysanthic prometatropy	$aB' \,.\, bA'$
	„ dysanthic hemi-prometatropy	$aB' \,.\, bC$
	„ synanthic exotropy	$aC \,.\, bC$
	„ dianthic exotropy	$aC \,.\, bC'$

It will thus be seen that there are 33 different forms of sibling relationship in plants corresponding to whole or half-brotherhood in horses, dogs or men; and further for a particular plant grown as a crop, although the nature of the plant may enable us to cancel certain of these relationships as impossible, there will generally be a considerable number left, and the proportions of each class may be quite unknown to us. For example, if we take seed from the same capsule on a plant whose flowers are capable of either cross- or self-fertilisation we may really have a mixture of nine different types of relationship owing to pollen from the anthers of the same flower, from anthers of other flowers of the same plant, and from anthers of flowers of different plants being scattered on or carried by insects to the same stigma or system of stigmata. Hence it is very difficult to compare the relationship of the bisexual offspring of some animals†, or even of the parthenogenetic offspring of certain insects with the relationship among offspring, which may range all the way from $aB \,.\, aC$ to $aA \,.\, aA$ in unknown proportions‡.

* While in every case the names lead to the same relationship symbolically expressed, the symbol may lead us to different names for the same relationship. Thus dis-ovarial synanthic exhomotropy = dis-ovarial synanthic hemi-hypermetatropy, etc., etc. We have reserved, however, the metatropic terminology for those special endotropic and exotropic relationships which cannot be expressed without it.

† Similar relations to those of plants may occur in animals having a repetition of gonads; but such animals have not yet been investigated from the standpoint of heredity, so that no comparison with our plant results is possible.

‡ If the individual flower, as is occasionally the case, possesses a system of ovaries or sub-ovaries (a_1, a_2, a_3, ...) and these are associated with individual systems of anthers (A_1, A_2, A_3, ...) then the system of relationships becomes still more complex and breaks up in the case of seed from the same flower head alone into the 37 types:

$$aA \,.\, aA = \begin{cases} a_1A_1 \,.\, a_1A_1;\ a_1A_1 \,.\, a_1A_2;\ a_1A_1 \,.\, a_2A_1;\ a_1A_1 \,.\, a_2A_2;\ a_1A_1 \,.\, a_2A_3;\ a_1A_2 \,.\, a_1A_2; \\ a_1A_2 \,.\, a_2A_1;\ a_1A_2 \,.\, a_2A_2;\ a_1A_2 \,.\, a_2A_3;\ a_1A_3 \,.\, a_2A_3;\ a_1A_2 \,.\, a_1A_3;\ a_1A_3 \,.\, a_2A_1; \end{cases}$$

$$aA \,.\, aA' = a_1A_1 \,.\, a_1A_1';\ a_1A_2 \,.\, a_1A_1';\ a_1A_1 \,.\, a_2A_1';\ a_1A_2 \,.\, a_2A_1';\ a_1A_3 \,.\, a_2A_1';$$

$$aA' \,.\, aA' = a_1A_1' \,.\, a_1A_1';\ a_1A_1' \,.\, a_2A_1';\ a_1A_1' \,.\, a_1A_2';\ a_1A_1' \,.\, a_2A_2';$$

$$aA' \,.\, aA'' = a_1A_1' \,.\, a_1A_1'';\ a_1A_1' \,.\, a_2A_1'';$$

$$aA \,.\, aB = a_1A_1 \,.\, a_1B_1;\ a_1A_2 \,.\, a_1B_1;\ a_1A_1 \,.\, a_2B_1;\ a_1A_2 \,.\, a_2B_1;\ a_1A_3 \,.\, a_2B_1;$$

$$aA' \,.\, aB = a_1A_1' \,.\, a_1B_1;\ a_1A_1' \,.\, a_2B_1;$$

$$aB \,.\, aB = a_1B_1 \,.\, a_1B_1;\ a_1B_1 \,.\, a_1B_2;\ a_1B_1 \,.\, a_2B_1;\ a_1B_1 \,.\, a_2B_2;$$

$$aB \,.\, aB' = a_1B_1 \,.\, a_1B_1';\ a_1B_1 \,.\, a_2B_1';$$

$$aB \,.\, aC = a_1B_1 \,.\, a_1C_1;\ a_1B_1 \,.\, a_2C_1.$$

If instead of collecting seed from a single capsule we collect all the seed from all the capsules of a plant, we may still further widen our range to 20 grades of sibship, and it is not by any means certain that these 20 grades will give as close a general relationship, as the 9 grades from the same capsule even if the plant and not the flower be the individual. For, the more capsules taken the more chance there is for variety in the crossing with other plants, i.e. there is increased admixture of the half-sibling relationships.

It is further obvious how wide must be the range of experiment if we are to determine the relative influence of these various modes of crossing; for, every heredity series to have validity ought to contain at the very least 100 plants, and the majority of the cases discussed require artificial fertilisation, and in some of the metatropic cases this is of a rather complex kind.

The series of experiments considered in this paper do not deal with the question of fertilisation at all, although tentative experiments in this direction have been commenced this year. Shirley poppies grown in masses are both self- and cross-fertilised, the latter principally by bees although the flowers are not honeyed. The relative extent of each form of fertilisation could probably not be determined without special investigation or experiment*. To what extent the cross-fertilisation is really endotropic and not exotropic, in the case of plants having sometimes a dozen flowers in blossom at the same time must also be the subject of a later enquiry. It is clear, however, that the relative proportions of homotropy, endotropy and exotropy may vary very largely from one crop to a second and thus largely influence the 'parental' and 'fraternal' relationships.

Other matters directly influencing the constants are, besides soil and climate, differentiation between early and late flowers and between high and low flowers (which is partly the same thing), and the effect of special treatment of individual plants. We shall refer to these topics again later.

(3) *Influence of Environment on Crop-constants.* It must be remembered that we had three different kinds of seed, and that two of these: Individual Plant Seed (α), and Individual Band Seed (β) were selected, and further the amounts of each subgroup sown in the different crops would naturally differ rather widely. Hence the mean and variability of any crop cannot be effectively compared with that of the first Hampden crop on the basis of either (α) or (β). For comparison the General Seed (γ) was provided, but unluckily this failed in two instances; at Bookham no return for (γ) was made, and at Highgate the experimental patch of (γ) produced only one or two starveling plants. We have the following results:

* Messrs Sutton & Co. to whom I am indebted for information on this point hold that the relative extent depends on the state of weather at the time of flowering and on other conditions.

TABLE I.

Control Seed (γ), 1900.

Crop	Capsules	Mean	S. D.
Original Hampden ...	4443	12·51	1·898
Highgate	wanting	wanting	wanting
Oxford*	531	9·29	2·134
Chelsea† II	40	10·45	1·688
Crockham	322	13·01	1·977
Bookham	wanting	wanting	wanting
Enfield I	403	14·26	1·753

Thus the town conditions of Chelsea and the potting at Oxford were unfavourable to the development of many stigmatic bands, while at Crockham and Enfield the soil or climate were more favourable than on the top of the Chilterns. The alterations in variability are not so marked, but the potting in a specially mixed soil while it lowered the mean seems to have increased the variability.

We can deal with the influence of environment in a somewhat different manner on the basis of the seed (α). We can compare what the weighted parents

TABLE II.

Weighted Parents and Local Offspring. Seed (α).

		Hampden Parents		Local Offspring	
Crop	No.	Mean	S. D.	Mean	S. D.
Original Hampden ...	706	12·68	1·219‡	—	—
Highgate	606	13·21	1·120	12·03	1·890
Oxford...	2728	12·72	1·106	10·76	2·293
Chelsea II	219	12·04	1·694	11·84	1·808
Bookham	1905	12·54	1·368	12·76	2·217
Enfield I	4204	12·76	1·165	13·95	1·622
		Enfield I, Parents		Local Offspring	
Original Enfield I ...	—	14·00	1·311§	—	—
Kidderminster	19,027	14·26	1·321	13·13	1·721

* The Oxford poppies were grown in pots placed on *gravel*. If we add to the above another 45 pots of seed (γ) grown in pots placed upon a *garden bed* we find for the whole series of 967 capsules: mean 10·87, S.D. = 3·001. This shows the great influence of an even slight change of environment.

† Chelsea I, the crop of 1899 from different seed gave on the basis of 1020 capsules a mean of 12·37 and a standard deviation of 1·680, the equality of the variabilities is possibly only accidental.

‡ These are the mean and standard deviation of the selected parent Hampden plants, of which I took all the seed from all capsules. A distinction must always be drawn between plant mean and capsule mean.

§ These are the mean and standard deviation of the selected 100 Enfield parent plants.

would have done under the Hampden conditions with what the offspring plants actually did in the given locality.

The mean of the 706 capsules on the Hampden parent plants was 12·75 with a standard deviation of 1·783, while the total Hampden crop had a capsule mean of 12·51 and standard deviation of 1·898. Thus my selected parentage had a higher mean and a much reduced variability, however measured. It will be observed, however, that the weighted parental mean, i.e. parents weighted with the number of offspring capsules, always differs from the mean of the parent plants. We cannot, however, determine how much of this was due to environment; the seed of the original 24 parent plants could not be distributed in normal population proportions, the offspring of each plant which survived differed in number according to the amount of seed sown, the thinning, etc. At Chelsea the storm chanced to destroy the offspring groups having parents near the mean, and thus artificially exaggerated the variability of the Chelsea parental selection. At Oxford where W. F. R. Weldon attempted to deal with individual plants by sowing in pots (and so by subjecting individuals to a like treatment a record of the relative fertility of the parent seed was possible) it was found that even the nature of the ground immediately under individual pots influenced individual development. Thus at Chelsea and Bookham, only 12 and 16 of the parents respectively contributed to the offspring, and this fully accounts for the variability of the actual weighted parents being in these cases considerably greater than that of the original Hampden parentage. The conditions at both Highgate and Oxford were adverse to the plant; at Highgate one capsule to a plant was about the average, and at Oxford the average was only 3·2. The low variabilities of the parentages at these places, are therefore probably due to the selection of parent seed by the environmental conditions, but a more definite statement than this it would be unwise to make. Comparing, however, the actual parentages with the local offspring we see at once that the Bookham and Enfield environments were more favourable to the development of stigmatic bands, and Oxford, Highgate and Chelsea less favourable than Hampden. The Kidderminster crop while far heavier than the Enfield in the number of fruit per plant shows a diminution in the mean number of stigmata. Thus the environment largely affects the number of stigmata, but there is no evidence to show that this is appreciably influenced by parentage in bulk. Thus while the selected parentages range only from 12·04 to 13·21, a range of 1·17, the resulting offspring means range from 10·76 to 13·95, or a range of 3·19. Nor is there any system in the arrangement, e.g.

Order of Parentage	Order of Offspring
Chelsea	Oxford
Bookham	Chelsea
Oxford	Highgate
Enfield I	Bookham
Highgate	Kidderminster
Kidderminster	Enfield I

which could be interpreted on the basis of a law of regression applied to the crop means.

Thus, whatever theory of heredity we apply, it must allow for (i) absolute variation of the mean due to treatment and environment of the offspring crop as a whole, and (ii) selection of parentages differing from crop to crop. Now these are the very points so often misinterpreted by those who fail to grasp the theory of regression. The first difficulty is got over by remembering that regression is not towards the parental but towards the filial mean, and the second by noticing that while both parental and filial variabilities and the correlation are changed by selection of parents, the slope of the regression line is not changed, and that the slope of this line would be the coefficient of parental heredity supposing the population to reproduce itself stably, i.e. without change of variation from generation to generation. In other words regression is summed up in:

Probable deviation of individual offspring from filial mean = coefficient of parental heredity × actual deviation of individual parent from parental mean.

It is this "coefficient of parental heredity," equal to correlation between parent and offspring multiplied by the ratio of filial to parental variability, that we must find to test how far heredity in our poppies is in accordance with what we have found it to be for men, horses, etc.

Lastly we may consider seed (β) drawn from a variety of capsules with the same number of stigmata, although we have here of course no measure of the original Hampden parentage. The following table gives the results.

TABLE III.

Weighted Capsules and Local Offspring. Seed (β).

Crop		Parent Capsules		Offspring Capsules	
	No.	Mean	S. D.	Mean	S. D.
Oxford...	1140	12·89	3·143	9·83	2·248
Chelsea II*	96	11·24	3·305	12·19	1·660
Crockham	2991	12·34	3·537	13·46	2·119
Enfield I	2066	12·27	3·319	14·26	2·018
Enfield II	566	14·41	2·203	12·16	1·751

It will be seen at once that the general results are the same, Oxford and Chelsea II are the poorest of the 1900 crops; Oxford is the most variable. But Enfield II was also a poor crop, the mean having fallen back in 1901 below even

* Chelsea II is hardly worth recording with only 96 capsules. 40 out of the 96 capsules obtained were on plants the seed of which was taken from capsules with 7 and 8 bands, whereas at Enfield only 360 out of 2066 capsules came from these low band groups. It is possible that low band seeds were better suited to the unfavourable Chelsea environment.

the Hampden parent value from which it rose in 1900. While in Table II. the variability of the offspring was in each case greater than that of the parents,—because we were comparing a capsule variability in the former, with a mean plant variability in the latter,—in Table III. the parent capsules are markedly more variable than the offspring capsules. This is precisely what we should expect, if the individuality on which heredity depends lies in the plant as a whole and not in the special capsule. We shall investigate this at greater length below.

(4) *Relative Fertility of Different Capsules.* Direct experiments were not made on relative fertility, but one or two points may be noted. In examining the Crockham capsules, we were again* struck by the sterility of the low banded, and to a lesser extent of the high banded poppies. W. R. Macdonell for the second Enfield crop selected 57 capsules and sowed from the seed of these capsules 57 rows. Of these rows 22 were from capsules with 9 to 12 bands, 28 from capsules of 13 to 15 bands and 7 from capsules of 16 to 20 bands. 10 rows of the first group, none of the second and 3 of the third were unproductive, or about 45 per cent. of the low banded and 43 per cent. of the high banded capsules failed, while none of the capsules near the modal value (about 14) were unproductive.

In the Oxford Band Series (β), W. F. R. Weldon sowed 110 pots from capsules 7 to 10, 150 pots from capsules 11 to 15, and 90 pots from 16 to 18, each pot being sown with a few seeds. 43 per cent. of the pots in the first group failed to show germinations, and 21 per cent. in the third group, while 8 per cent. only failed in the group from 11 to 15, the modal value of the original Hampden crop being 12·75 bands per capsule. The poppies were sown on March 25—26, and the productive pots counted from April 12 to 20. Further, on April the 21st, the first group from capsules of 7 to 10 had about 5 seedlings per pot, the second group from capsules of 11 to 15 about 22 seedlings per pot, and the third group from capsules of 16 to 18 had 10 seedlings per pot.

Now of course this seedling result admits of more than one interpretation, for it may be said that there was more seed of the modal capsules and that more of it may have been sown. While it required comparatively few modal Hampden capsules to provide a good amount of seed, almost every available low-banded and many high-banded had to be dealt with to collect enough seed at all from these classes of capsules, thus the seed was certainly not in proportion to the frequency of the capsules in the original crop, nor was it sown in that proportion. Hence Weldon's percentage of productive pots and proportion of seedlings seem on the whole to confirm Macdonell's results, which started not with equal quantities of seed, but with the seed of *single* capsules.

From the Seed (α) it is difficult to draw any conclusions, for we do not know what proportion of the seed came from modal capsules, and it is therefore not possible to test the general principles (i) that the modal capsule has most seed

* *Grammar of Science*, p. 444.

and (ii) that this seed more readily germinates. On the whole there is evidence enough in favour of these principles from the band series to make it desirable that experiments directly bearing on these points should be instituted next year.

(5) *Homotyposis.* We have now eight cases of homotyposis worked out for Shirley Poppies. The results obtained show a very considerable variation in the homotypic constant, and it is undoubtedly affected by the environment and treatment of the crop. Chelsea II was such a failure that no further constants were determined for it. The Highgate crop had very rarely two fruits to the plant and so homotyposis could not be dealt with. Both the Bookham and the Crockham crops were gathered as plants which were tied into bundles for each series, the other crops having the fruits collected off each individual plant into separate receptacles. Thus in the former crops all the capsules from the same series, but possibly not all the capsules from the same plant, have been grouped together. This would to some extent affect the homotyposis of these series*. The Chelsea I crop suffered, as has been elsewhere recorded, much from selection after gathering and the Enfield II crop was a remarkably poor one, having on the average only 2 to 3 capsules per plant. The general results are given in Table IV.

TABLE IV.

Homotyposis in Shirley Poppy.

Crop	Year	Plants	Capsules	Capsules per plant	Pairs	Mean	S. D.	Homotyposis
Hampden ...	1899	176	4443	25·2	197,478	12·61	1·885	·5238
Chelsea I ...	1899	325	1020	3·1	2,756	12·37	1·680	·6149
Oxford ...	1900	861	2728	3·2	11,588	11·39	2·173	·5573
Crockham...	1900	355	2991	8·4	61,334	13·39	2·055	·3446
Bookham ...	1900	1029	1905	1·9	4,338	13·14	2·330	·3475†
Enfield I ...	1900	907	2066	2·3	19,370	13·78	1·518	·5093
Enfield II ..	1901	239	566	2·4	1,244	12·72	1·717	·6831
Kidderminster	1901	1618	19,110	11·8	329,840	13·18	1·702	·4025
Mean	—	—	—	—	—	—	—	·4978

Now it will be seen at once from this table that the magnitude of the homotyposis is not related at all or not in any simple manner to number of plants, number of capsules, number of pairs, or to average number of capsules per plant used in its determination. Nor does a high or low variability seem significant for changes in the homotypic correlation. On the other hand the four crops with

* G. U. Yule writes: "The plants were sometimes broken, thus capsules belonging to the same poppy may be separated, but capsules entered between bars certainly belong to the same plant." I.e. one broken plant might occasionally be counted as two plants.

† The Bookham homotyposis is unsatisfactory; the record of capsules from the same parentage was satisfactory, but that of capsules from the same plant was very defective, as a rule only 1 to 3 capsules were recorded as certainly from same plant, but two plants had between 30 and 40 capsules, and these had to be omitted, or they would have given rise to one-third the total number of pairs.

low means have the four highest correlations and the four crops with high means have the four lowest correlations. But even thus the order of correlations is by no means the order of relative means. We hold that, at present, all we can safely conclude is that any treatment or change of environment of a crop which tends to raise its mean, will in this case lower its homotyposis. Perhaps we might even say that 'starveling' conditions, which certainly existed in Chelsea I, Oxford and Enfield II, tend to intensify the homotypic relations, while favourable conditions such as those of the magnificent Kidderminster crop, or the very good Crockham crop tend to reduce the homotyposis (see p. 58).

The general average ·498 of this series of eight agrees excellently with that ·499 found for a former series of thirty-seven cases in the animal and vegetable kingdoms*—only the first two crops being common to the two series. We see that treatment may considerably influence individual results, but, as it appears as likely to tend in one as the other direction, the mean remains steady.

(6) *Methods of measuring Plant-character for Inheritance.* When we are considering quantitative inheritance in plants we are at once confronted by the fact that although the flower, fruit, or leaf may be convenient subjects for observation, they are not as a rule unique in the plant, and consequently we are confronted with a multiplicity of elements for the determination of the character—a very little observation showing that variability within the individual is nearly as marked as the variability of individuals among themselves. Thus a single fruit or flower cannot be taken as a sample character of an individual,—for doing so is at once assuming that homotyposis is perfect or unity, which we know is very far from the case. The consideration therefore of plant inheritance leads us at once owing to the multiplicity of organs to homotypic relations. A like result rarely occurs when we deal with heredity in mammals, for their homotypes are not organs, which directly present themselves as suitable for the problem of inheritance. In dealing with plants accordingly we cannot take a single organ, but must deal either with all such organs, or at least with a sufficiently large random sample of them.

If we express the character of the individual plant by the mean of any quantity observed or measured on such organs, we are at once impressed with the varying degree of weight to be given to this character according to the number of fruits or flowers, etc. upon which the estimate is based. A Shirley poppy may have any number of capsules from one to eighty and the average number of stigmatic bands per capsule, which may be taken to express the individual plant character, will have a great variety of weights according to the number of capsules dealt with. In some plants it might be possible to take a random sample of 25 or 50 flowers or fruits, etc. and strike the average of these. But this would be laborious in the case of a crop of 1000 individuals. 5 to 10 as a sample would be easier, but not nearly so reliable, and even so few could not have been obtained in the case of many Shirley poppies. We therefore determined to take all the capsules and

* *Biometrika*, Vol. I. p. 343.

include them all in our calculations, but, to avoid the great labour of taking means and weighting, to adopt other processes for linking parents to offspring, using the mean of the capsules of the former and the individual capsule of the latter.

Let us consider the N offspring plants $o_1, o_2, o_3 \ldots$ of one individual parent plant with capsule-mean P, and let these offspring plants have $n_1, n_2, n_3 \ldots$ capsules; suppose these capsules to have $c_1, c_1', c_1'' \ldots c_2, c_2', c_2'' \ldots c_3, c_3', c_3'' \ldots$ stigmata respectively.

Then the mean character of an offspring would be $o_1 = S(c_1)/n_1$, and the offspring mean, if all individuals were of equal weight, would be

$$O = S(o_1)/N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}.$$

If O were plotted to each P, we should have a series of points, from which we could at once deduce the regression line of offspring on parents, and so the intensity of heredity.

If the individual offspring have not equal numbers of capsules, the offspring means ought to be weighted, and we should have instead of (i)

$$O = S(o_1\sqrt{n_1})/S(\sqrt{n_1}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}.$$

This formula is extremely troublesome for practical calculations, and when we are dealing with perhaps 50 to 100 offspring of each parent practically unworkable.

If we weight, however, with n and not $\sqrt{n}$, we have

$$O = S(o_1 n_1)/S(n_1)$$
$$= \frac{\{S(c_1)/n_1\}\, n_1 + \{S(c_2)/n_2\}\, n_2 + \{S(c_3)/n_3\}\, n_3 + \ldots}{n_1 + n_2 + n_3 + \ldots}$$
$$= \text{mean of all capsules in the array of plants due to one parent} \ldots \text{(iii)}.$$

In this case all we have to do is to correlate the individual capsule with the parental mean capsule; this correlation will not be significant for heredity, but if we calculate the slope of the corresponding regression line, it will be the slope of the regression line of parental inheritance, as far as it is legitimate to replace (ii) by (iii). Direct test in a few cases showed that with a considerable number of offspring, not only (ii) and (iii), but even (i) led to very close results.

Accordingly our first method will consist in correlating all offspring capsules with the parental mean capsule, and then determining the slope of the regression line—the measure of heredity, free, as we have seen on p. 65, from the effects of selecting individual parents, and also of environmental change of mean.

Secondly we may obtain a measure of the intensity of inheritance in the following manner. We may compare the average variability of an array of offspring due to a single type of parent plant with the variability of all the offspring population. This method has certain advantages, if we suspect that the environment of a crop has not necessarily been continuous throughout, and that

sub-environments may have altered in an arbitrary manner the means of the different parent plants. For example, with only a few parents some may have been more highly favoured by light, soil or water than others. At any rate this method is valuable for purposes of control, although as it involves the labour of finding individual offspring means, it can only be occasionally applied.

Thirdly we may proceed by the homotypic relationship. This requires a brief theoretical treatment.

Let p be the mean character in any individual parent, P be the mean parent; o the mean character in any offspring, O the mean offspring and R parental correlation; then if σ_p σ_o be the standard-deviations of parent and offspring respectively, and N the total number of cases, we have

$$R = \frac{S(p-P)(o-O)}{N\sigma_p\sigma_o} \quad \text{.....(iv).}$$

But if we weight the offspring as on p. 69 with the capsule numbers of each individual, we shall have

$$R = \frac{S(p-P)(no-nO)}{N'\sigma_p'\sigma_o'},$$

where n = number of capsules in an offspring plant, $N' = S(n)$, σ_p' = standard deviation of parents and σ_o' of offspring weighted with capsule numbers.

If now we use c to denote the capsule, we have $no = \Sigma(c)$, where Σ is the summation for every capsule of an individual. Further

$$O = S(no)/S(n) = S\Sigma(c)/N = C,$$

where C is the mean capsule of the whole series of offspring. Hence we find

$$R = \frac{S(p-P)\Sigma(c-C)}{N'\sigma_p'\sigma_o'}$$

$$= \frac{S'(p-P)(c-C)}{N'\sigma_p'\sigma_o'} \quad \text{.....(v),}$$

where S' is a summation of every parent plant and offspring capsule. But if r be the correlation between parent plant and offspring capsule

$$r = \frac{S'(p-P)(c-C)}{N'\sigma_p'\sigma_c}.$$

Hence it follows that

$$R = \frac{\sigma_c}{\sigma_o} r^* \quad \text{.....(vi).}$$

* Multiplying by σ_o'/σ_p' we have the regression coefficient equal to $r\sigma_c/\sigma_p'$ the result of our first method.

But since the present system of weighting makes O and C the same, we have

$$\sigma_c^2 = s_c^2 + \sigma_o'^2,$$

where s_c is the mean S.D. of all the offspring arrays of capsules $= \sigma_c\sqrt{(1-\rho^2)}$, if ρ be the coefficient of homotypic correlation. Thus we have

$$R = r/\rho \quad \text{.....(vii).}$$

(vii) would probably be a good result to work from if we wanted to find the parental heredity R from r and ρ in material where neither parents nor offspring were selected, and where there was no marked change of environment between the two generations. Possibly the influence of selection of offspring and of their environment, since they affect both r and ρ, may be less marked in (vii) than in $r\sigma_c/\sigma_p'$, the regression given by the first method, for r and ρ (see Table VI.) may tend to rise and fall together, and thus the third method may in some cases give us better results than the first.

TABLE V.

Parental Inheritance. First Method.

Crop	Correlation of Parental Mean and Offspring Capsule	Slope of Regression Line
Highgate ...	·3230	·5451
Oxford ...	·1960	·4064
Bookham ...	·2199	·3412
Enfield I ...	·1864	·2595
Kidderminster	·1220	·1589
Mean	·2095	·3422

(7) *Results for Parental Inheritance.* We have seen that the first method ought to eliminate the effect of a selected parentage, but that it would not be uninfluenced by a selection of offspring. There can hardly be a doubt that the order of Table V. is practically that of the stringency of environment for the five crops,—the most starveling crop being the Highgate one and the most flourishing the Kidderminster. Or, it would seem that the more luxuriant the crop, the less intense is the strength of heredity. The mean value of the slope of the regression line is not far from the $\frac{1}{3}$ originally given for parental heredity in human stature by Galton, but it is considerably less than the value (about ·45) recently obtained for about sixteen cases in man*. We have already pointed out that the restriction of the variability of the general population to be found in an array of offspring due to a single parentage might still be maintained, if different parents had received different treatments. With the assistance of Marie A. Lewenz this point was investigated. She struck the means of all the capsules on each of the 907 Enfield I plants. These plants were then grouped into families and their

* Unpublished Family Measurements on upwards of 1000 families in the possession of K. Pearson.

individual standard deviations as families taken, as well as the mean and standard deviation of the population of 907 plants. The mean of the plant means was 13·75 stigmata, and their standard deviation 1·370. The average variability (or S.D.) of plants from the same parentage was 1·272, and the weighted mean square deviation—i.e. $\sqrt{S(n\sigma^2)/N}$, where σ is the standard deviation of a fraternity of n, and N the total number of plants—was 1·287. If r be the parental correlation $\Sigma\sqrt{1-r^2}$, Σ being the total plant variation, should be that of an array of offspring. Hence equating this to 1·287 we find $r = ·3427$. Had we equated it to 1·272 we should have found $r = ·3716$. In either case this, our second method, gives a value much closer to that ·3660 of the third method, than the value ·2595 found for the slope of the Enfield crop regression line.

Finally to illustrate the second method more completely a correlation table has actually been formed for the means of parent and offspring plants in the case of the Enfield I crop. The value of the parental mean plant is 12·69 of the offspring plant, 13·75, of their respective standard deviations 1·199 and 1·370; the correlation is ·1561 and the regression coefficient of offspring on parent plant ·1784. Nor is this result really to be much wondered at, for not only are the means of the stigmatic bands much influenced by the number of capsules on the plant, but the flowers that come out early in the season have fewer bands than those which come later. Thus taking the Oxford record for *first* flowers on 322 plants, we divided them into 161 earlier flowers of each family and the 161 later flowers, and found a correlation between the number of stigmatic bands of the first flower and its lateness in coming out of ·2078. The number of capsules to the individual plant, and the dates at which it produces them, tend to obscure the influence of pure heredity, and make the stigmata however easy to count and deal with a by no means ideal character to study heredity upon.

To avoid the difficulty due to differentiation in capsules, a correlation table was formed for 327 *first* flowers and the means of the parent plants in the Oxford crop. We found parental mean 12·74, standard deviation 1·240; stigmata of first flowers of offspring 11·02, standard deviation 1·828; correlation ·2438, and regression of offspring first capsule on parental mean = ·3594. There can be small doubt that, if we had compared the stigmata on the first flower of the parent with those on the first flower of the offspring, we should find heredity in poppies as intense as in forearm or cephalic index for the case of man. It is true that the Enfield I and Kidderminster crops show a remarkably low parental correlation, but both were large crops and it is highly probable that the environment of the strips devoted to special families was to some extent differentiated, and this may well have weakened the apparent strength of heredity*.

Finally if we attempt to obtain the parental correlation by our third method, we have the following table:

* Mr John Notcutt assures me that for the Kidderminster crop there was no differentiation in environment apparent for either light, shelter or soil.

TABLE VI.

Parental Inheritance. Third Method.

Crop	Correlation of Parental Mean and Offspring Capsule	Homotyposis	Parental Correlation
Highgate ...	·3230	[·6490 ?]	[·4976 ?]
Oxford	·1960	·5573	·3517
Bookham ...	·2199	[·3475 ?]	[·6328 ?]
Enfield I ...	·1864	·5093	·3660
Kidderminster	·1220	·4025	·3031
Mean... ...	·2095	·4897 [·4931 ?]	·3403 [·4302 ?]

The Highgate poppies were a starveling crop with rarely more than one capsule to the plant. Hence no determination of homotyposis was possible. The most like crops were Chelsea I and Enfield II and the mean homotyposis of these two is put in brackets with a query. We have already seen that the Bookham homotyposis result (p. 67, ftn.) is doubtful, and probably this result ought also to be excluded. Omitting these two results we find the mean in good agreement with that obtained from the first method, i.e. ·3422.

We may safely conclude that parental relationship for the stigmatic bands in Shirley poppies is expressed by a correlation lying between ·35 and ·4, but that its value is considerably influenced by the conditions of the individual crop, or even by differential treatment of parts of it.

(8) *Hereditary Influence of the Capsule and of the Plant.* One of the questions proposed to those assisting in the present investigation was the problem of the relative individuality of the plant and its fruit. Was the resemblance of the offspring to the special capsule from which its seed had been extracted greater than to the average capsule of the parent plant? The data for answering this question were to be found in the crops grown from Seed (β). Only three such crops were sown in 1900 and the following table contains the results.

TABLE VII.

Capsule Parentage.

Crop	Correlation of Offspring and Parent Capsules	Regression	
		Offspring on Parent Capsule	Offspring on Parent Plant
Oxford	·1220	·0873	·4064
Crockham ...	·1949	·1168	— *
Enfield I ...	·1492	·0907	·2595
Mean... ...	·1554	·0983	·3320

* No crop was grown at Crockham from Seed (a).

It would seem accordingly that individuality lies in the plant rather than in the fruit, i.e. that an offspring capsule only takes after the parent capsule, because that capsule is grown on the parent plant, and that the individuality of the capsule in the plant does not influence the seed it bears*.

The seed of the above crops was all of the same kind, namely "capsule parentage 14" consisted of all the seed contents of many 14-capsules on *a variety* of Hampden plants. W. R. Macdonell in a crop of 1901 modified this experiment to some extent by selecting the seed of 57 capsules, of which 44 gave rise to 239 plants with 566 capsules—a 'starveling' crop. Thus while the plants at Hampden had an average of 25·2 capsules per plant and this gave a wide variety of capsules on the plant, the Enfield I had only 2·3 capsules on the plant and the Enfield II, the above 239, only 2·4 capsules per plant. The individuality of the plant in Enfield II was thus possibly more likely to be represented by a selection of its few capsules than by a selection from the many capsules on a Hampden plant. The results obtained by Macdonell were:

Crop	Correlation of Offspring and Parent Capsules	Regression of Offspring on Parent Capsule
Enfield II ...	·2819	·2241

The regression here is more than double that found in the earlier series of experiments, but it is still considerably less than the average value of the regression of offspring plant on parent plant, ·3422, determined on p. 73. Macdonell further correlated the capsules of the Enfield II crop with the grandparental Hampden plants,—grandparental means and offspring capsules. He found for the weighted grandparental mean of the 566 offspring 13·11 and for the grandparental S. D. ·767. This shows that much selection of grandparents had taken place for the mean and S. D. of the Hampden parents were 12·68 and 1·219. The correlation of grandparent plant and offspring capsule was ·0066 and the regression of the former on the latter ·0190. Both of these are really insignificant. But it is difficult to appreciate in this case the allowance which has to be made (i) for selecting in the intermediate stage not a parent plant, but an arbitrary capsule off the parent plant, and (ii) for comparing offspring capsule with grandparental plant. The result as far as it goes confirms of course the previous statement, that the closer degree of hereditary relationship is between plant and plant and not between capsule and plant ancestry.

That a slight variation in the material dealt with makes a considerable difference in plants, which are peculiarly susceptible to environmental and seasonal

* This conclusion has been further confirmed by the case of *Nigella Hispanica*. There was sensibly no relationship in this plant between the segmentation of the individual capsule on a plant and that of the capsule from which the seed which gave rise to that plant was taken. K. P.

influences in their flowers, is well evidenced by the following results provided by W. F. R. Weldon from his Oxford crop.

TABLE VIII.

Nature of Group dealt with	Correlation of Parent Plant and Offspring Capsule	Regression of Offspring on Parent Plant
(*a*) Early capsules (apical flowers) of principal plants*	·2323	·4003
(*b*) Capsules on plants—not starvelings, i.e. with at least three capsules	·2430	·4050
(*c*) All capsules on principal plants...	·2295	·4295
(*d*) All capsules on all plants	·1960	·4064

The regression therefore is fairly constant, although the correlation varies. The relationship as expressed by the correlation is greatest, when we avoid differentiation in either the plants themselves or in their fruits, but such differentiation does not widely influence the regression, which for this Oxford crop takes a mean value not very divergent from what has been found for the forearm in man (·42 as mean of four results).

(9) *Grandparental Inheritance.* Our data for grandparental inheritance are very limited and suffer even more than the parental material from changes in crop treatment. Thus the original grandparents were grown at Hampden on the top of the Chilterns, the parents at Enfield and their offspring at Kidderminster. There is thus considerable change not only in soil but treatment; unfortunately we had not the means of carrying on the experiments under uniform supervision in the same environment. From the Kidderminster crop, we had 19,204 capsules due to 100 parent plants and 24 grandparental plants—in all cases the pollen plants being unknown. The grandparental plant mean was 12·64 and the standard deviation 1·1609. The grandchild capsule mean was 13·19 with a standard deviation of 1·7526. The correlation between grandparental plant mean and grandchild capsule was ·0490. The regression of offspring capsule on grandparental plant mean was thus ·0739. If we use the third method and the value of the Kidderminster homotyposis it would be ·1216. The only result with which these can be compared is the Enfield II grandparental capsule result (see p. 74). Dividing this by the product of the two Enfield homotypic values, we should have ·1897 for the grand-

* On the thinning out one plant, the principal plant was left in a pot, but later other seed germinated and small secondary poppies were found in a number of pots.

parental relationship of plant means. Our meagre results may then be summed up in the following table:

TABLE IX.

Grandparental Inheritance.

Crop	Method	Grandparental Inheritance	Parental Inheritance
Kidderminster	Regression	·0739	·1589
"	Homotyposis	·1216	·3031
Enfield ...	Homotyposis	·1897	·3660

These results at least indicate that, however determined, the grandparental relationship is fairly *close to half the parental.* It is much to be regretted that we have not wider data, especially on a less luxuriant crop than the Kidderminster one, for the above results are very variable.

(10) *Colour Inheritance.* W. R. Macdonell made an elaborate classification of the colour of 1604 flowers divided according to their parentage into 24 groups. He formed 13 colour classes, which after consideration were classified into three groups according to the intensity of red colouring matter in the flower. The first group embraced all the range from dark red to red-whites; the second the pure pinks and the third all the pink-whites to pure whites. In this classification attention was not paid to the base of the petal, but to the middle portion and margin. The following are the frequencies of the original scale:

(1)	Red, bordered lilac ...	2	(8)	Pink-white, bordered pink	0
(2)	Red	275	(9)	Pink-white	174
(3)	Red, bordered white ...	70	(10)	Pink-white, bordered white	155
(4)	Red-white	31	(11)	White, bordered pink ...	0
(5)	Red-white bordered white	43	(12)	White faint pink	76
(6)	Pink	622	(13)	White	50
(7)	Pink, bordered white ...	106		Total	1604

Classes (8) and (11) had representatives in crops from Seed (β) and (γ). Classes (1) to (5) formed our first general class, (6) our second, and (7) to (13) our third. This grouping enabled us to use the method for dealing with characters not quantitatively measurable, assuming that the distribution of red colouring matter is an approximately normal one. If σ be the standard deviation of the whole population of 1604 offspring flowers, and Σ the mean standard deviation due to the arrays having a common parent $\Sigma = \sigma\sqrt{1-r^2}$, where r is parental correlation. Now σ was found in terms of p, the pink range, by means of tables of the probability integral, and we have $\sigma = p/(1{\cdot}0218)$. The standard deviation

of each array of offspring was found in like manner, and the pink range eliminate between this and the racial standard deviation. There resulted

$$\Sigma/\sigma = \cdot 9111,$$

or Parental Correlation in Colour = ·4122.

This result,—in excellent accordance with those for forearm and span in 8 cases of 1000 each in man—suggests that colour, even with the rough classification indicated, may be effectively dealt with. The continuity of the intensity of colouring in Shirley Poppies is fairly obvious when crops of thousands of flowers are examined. We propose next year to form a more elaborate scale of colouring and continue the experiments especially on the colour side.

Meanwhile the only comparable data we can use occur in the Oxford crop. Here the colours were observed on the first flowers of 319 plants distributed among 24 parentages. Owing however to a different colour classification to W. R. Macdonell's, only 11 of these parentages can be used,—the colour scale not extending over three classes in the others. The mean standard deviation of these 11 parentages—containing only 153 flowers, however,—bears the ratio of ·8479 to the standard deviation of the whole set. Hence we find parental correlation ·5302. This result is not very reliable, of course, the data being so few; yet it is in accordance with the higher parental inheritance values obtained generally for the Oxford crop.

(11) *Fraternal Correlation.* While for resemblance of offspring to parent we have compared an array of offspring plants with the mother plant the seed being taken from one or all capsules of the mother plant, we have a less definite conception in the case of an array of brother plants. All members of the array have, of course, a common mother plant, they are co-ovarial or bi-ovarial brethren, but the proportions of synanthic, dianthic and dysanthic crossing are quite unknown. Hence it is difficult to guess *a priori* at the amount of common pollen parentage in pairs of brother plants. We can hardly, however, suppose that all the seed in the same capsule has the same pollen parentage, but, except in the case of self-fertilisation, we should expect seed from the same capsule to give plants more alike than seed from different capsules. Taking the best results for fraternities of common parentage in man—mean of about 30 results—we should expect the resemblance of brothers to be about ·5. This might be reduced to anything down to ·25, i.e. case of half-brothers, by the degree of diversity in the pollen, from which the arrays of brothers are due. Hence we must be prepared for fraternal correlation in poppies lying between ·25 and ·5, approaching the lower limit, if there has been the freest possible cross-fertilisation, i.e. a great variety of pollen carried not only to the same mother plant, but even to individual ovaries on that plant.

Before we consider actual numerical results we must, however, modify them for homotyposis, remembering that our tables have been formed by comparing

fraternal capsules and not fraternal plants,—at least in the case of stigmata as character.

Let r = fraternal correlation of plant means and R = correlation of capsules on brother plants, ρ = homotypic correlation. Let Σ = a summation within a fraternity and S throughout a population.

Let z = number of stigmata on a capsule, and m the mean number of stigmata on a plant of n capsules; let a bar over a letter denote the mean value of that quantity for the whole population and subscripts 1 and 2 refer to a pair of brother plants. Then

$$z = m + x$$

$$\left.\begin{aligned} \Sigma(z) &= nm \\ S(z) &= S(nm) \end{aligned}\right\} \text{ and } \bar{z} = \bar{m},$$

where $\bar{m}$ is the weighted mean of the plant means.

$$z^2 = m^2 + 2mx + x^2,$$

$$\Sigma(z^2) = nm^2 + \Sigma(x^2) = nm^2 + n\sigma_x^2,$$

if σ_x^2 = standard deviation of an array of capsules on the same plant.

Hence σ_z being the standard deviation of all capsules

$$\sigma_z^2 = \frac{S(z^2)}{S(n)} - \bar{z}^2 = \frac{S(nm^2)}{S(n)} - \bar{m}^2 + \frac{S(n\sigma_x^2)}{S(n)}.$$

The first two terms on the right-hand side form σ_m^2, the standard deviation squared of the means, the third term is the mean square of the standard deviations of the arrays of capsules on the same plant $= \sigma_z^2(1-\rho^2)$, if ρ be the homotypic correlation. Hence we have

$$\rho^2\sigma_z^2 = \sigma_m^2.$$

Next

$$\begin{aligned} R\sigma_z^2 &= \frac{S(z_1 z_2)}{S(n_1 n_2)} - \bar{z}^2 \\ &= \frac{S\{n_1 n_2 m_1 m_2 + m_2 n_1 \Sigma(x_1) + m_1 n_2 \Sigma(x_2) + \Sigma(x_1)\Sigma(x_2)\}}{S(n_1 n_2)} - \bar{m}^2 \\ &= \frac{S(n_1 n_2 m_1 m_2)}{S(n_1 n_2)} - \bar{m}^2 \\ &= r\sigma_m^2. \end{aligned}$$

Therefore

$$r = R/\rho^2.$$

Here of course

$$\bar{z} = \frac{S(n_2 z_1)}{S(n_1 n_2)} = \frac{S(n_1 z_2)}{S(n_1 n_2)},$$

$$\bar{m} = \frac{S(n_1 n_2 m_1)}{S(n_1 n_2)} = \frac{S(n_1 n_2 m_2)}{S(n_1 n_2)},$$

and it is assumed that the standard deviations about these weighted means of the capsules weighted with the number of their brother capsules, will be closely equal to the standard deviation of the capsules weighted only with number of capsules in the plant. This will be closely true if the number of capsules per plant, and the number of plants in the fraternity are not widely divergent as for example they were in the case of the Bookham crop*.

TABLE X.

Stigmatic Bands. Fraternal Correlation.

Crop	Number of Plants	Number of Capsule Pairs	Capsule Correlation	Homotyposis	Fraternal Correlation
Highgate ...	606	46,490	·2513	—	·2513
Oxford ...	861	338,276	·0898	·5573	·2891
Enfield I ...	907	706,652	·0496	·5095	·1912
Kidderminster	1618	3,813,832	·0740	·4025	·4568
Mean ...	—	—	—	—	·2971

The Highgate crop had with rare exceptions one flower only per plant. Homotyposis could not therefore be considered. We are really comparing as fraternal character the first or apical flower on each plant. This was also done, as will be seen later, at Oxford with the result that the correlation was ·2561, a result agreeing excellently with that from Highgate. The most divergent result is that for the Kidderminster crop which was also the most anomalous in the parental relationship†. While the fraternal relationship is over-emphasised, the parental was diminished—these results are precisely what we might expect if the plants due to different parentages received differential treatment, i.e. if the environment varied somewhat from sub-crop to sub-crop. In this case the fraternal correlation would be emphasised by the differential environment, which would at the same time tend to distort, and so diminish the parental regression. The Kidderminster crop was such a large one—there were 100 separate parentages, that it would be almost impossible to avoid unrecognisable differentiation.

The average of the above results is of course very satisfactory. The deduction, however, indirectly of the fraternal correlation by aid of the homotypic coefficient may be objected to. Especially is this the case, when we take crops in which there is great variability in the number of capsules to the plant and of brother plants to the family. We here reach the same objection ultimately as arises from endeavouring to form the capsule mean for each plant, this mean having to be based on sometimes 1 or 2 capsules and on other occasions on 30 to 40 capsules. So that unless one weights individual means, one may reach very discordant

* The Bookham crop could not be considered for the fraternal data, because some of the plants being divided into two, we could not distinguish homotypic from fraternal relationship.

† Possibly the Enfield I crop was largely self-fertilised.

results. The weighting of 30,000 to 50,000 pairs of brother plants is not, however, a task to be lightly undertaken*.

Accordingly it is of interest to be able to turn to W. F. R. Weldon's Oxford observations on the *first* flowers of 320 odd plants. He observed the following characters on these first or principal flowers:

(*a*) the number of stigmatic bands,

(*b*) the number of petals,

(*c*) the petal length,

(*d*) the breadth of the margin, when coloured differently from the body of the petal,

(*e*) the extent of basal patch,

(*f*) the intensity of wrinkling on the petal,

(*g*) the colouring on the middle part of petal.

For (*b*), (*d*), (*e*), (*f*) and (*g*) Alice Lee worked out the fraternal correlation by the methods of the memoir on the inheritance of characters not quantitatively measurable†. (*a*) and (*c*) were worked out in the usual way by the long form of correlation table.

Under (*b*) the classes were four petals, four petals + one or more petaloid stamens and more than four petals. The ultimate division of the table for purposes of calculation being made at the normal four petals, and the second group containing everything from a single additional petaloid stamen up to cases of 20 to 28 petals. The classes under breadth of margin were: broad (*b*); broad-slight (*bs*),—one petal pair broad, the other slight, one case only; slight (*s*); slight-none (*ns*), one petal pair slight, the other none, five cases only; and none (*n*). The resulting correlation table was worked out for two divisions and the average taken.

The extent of basal patch was perhaps a less satisfactory character to estimate; the groupings were: none (*n*); none to slight (*ns*); slight (*s*); slight to well-defined (*sd*); well-defined (*d*); well-defined to large (*dl*); and large (*l*), understanding by this last category a large indefinite patch passing right up into the body of the petal. The wrinkling of the petals judged about the same period in each case after opening, twelve hours, was divided into: frilled (*f*); wrinkled (*w*); slightly wrinkled (*sw*); slight to no wrinkling (*sw* to *nw*); and no wrinkling (*nw*), the intermediate class being chiefly due to flowers with diversity between the two pairs of petals. Lastly the colour appreciation was based on the middle third of the petal and the classes adopted in grouping W. F. R. Weldon's much more detailed descriptions were: red (*r*); red-pink (*rp*); pink (*p*); pink-white (*pw*); pink-white-white (*pw. w*) and white (*w*), the intermediate classes (*rp*) and (*pw. w*)

* The Enfield I crop gave 34,486 pairs of brother-plants and the Kidderminster crop must have had between 50,000 and 60,000 pairs.

† *Phil. Trans.* Vol. 195, A, p. 1.

having only 7 and 6 representatives respectively and being due either to diversity in individual petals, or to actual transition cases, which could be only thus classified. In all these cases the actual continuity of the character observed was easy to demonstrate, and there was no hesitation in applying the methods of continuous variation. The determination of the dividing lines for purposes of calculation was made in general for 2 or 3 different groupings and the mean result appears in Table XI.

TABLE XI.

First Flower on Oxford Poppies. Fraternal Correlation.

Character	Number of Fraternal Pairs	Correlation
Stigmatic Bands	4716	·2561
Number of Petals	4692	·2256
Petal Length	4480	·2767
Breadth of Margin	4524	·1876
Extent of Basal Patch ...	4716	·2215
Intensity of Wrinkling ...	3912	·2149
Colour of Middle Third ...	4602	·3401
Mean	—	·2404

If we add to these the Oxford result of the previous table for the stigmatic bands of all capsules, i.e. ·2891, we have for the average relationship of pairs of brothers in the Oxford crop ·2514,—a result in excellent accordance with what we might expect from the case of man (·5), if we allow for most complete cross-fertilisation in the original Hampden crop*.

Lastly W. F. R. Weldon made two other measurements on some 600 Oxford plants, (i) the height above ground of the node from which the terminal flower was given off, (ii) the height of the terminal seed-capsule from the ground. A. Lee has found the fraternal correlation for these two characters, which are the only plant as distinguished from pure flower characters dealt with.

TABLE XII.

Oxford Crop. Fraternal Plant Correlations.

Character	Mean Value	S. D.	Number of Brother Pairs	Correlation
Nodal Height ...	27·53 cms.	9·353 cms.	13,800	·4123
Capsule Height ...	51·58 cms.	12·644 cms.	13,800	·3782
Mean	—	—	—	·3952

* 1899 was a remarkably fine dry summer and the bees were at work on the Hampden poppy crop all day long, day by day.

These results are high, approaching more nearly the result of the Kidderminster crop. But plant height is a character peculiarly influenced by differential environment, and in some fraternities the proportion of secondary poppies in the pots was far greater than in others. Hence we can hardly lay much stress on these results, which may be largely emphasised by the overcrowding of some of the fraternities as compared with others.

Capsule Fraternities, or Co-ovarial Brotherhood.

W. R. Macdonell's observations in the second Enfield crop, in which seed from the same capsule only was used for fraternal arrays, enable us also to form another fraternal correlation based not on common plant, but on common capsule parentage. The mean of the capsules was 12·39 and their S. D. 1·782, and we have the following results.

TABLE XIII.

Brothers from same Capsule and from same Plant.

Crop	Capsule Correlation	Homotyposis	Plant Correlation
Enfield I ...	·0496	·5093	·1912
Enfield II ...	·1764	·6831	·3780

We thus reach the conclusion that plants from seed of the same capsule will be nearly twice as closely related to each other as plants from seed of all capsules on the same plant. But on the other hand plants from the same capsule will be less like the parent plant, than plants from all capsules of the parent plant. The explanation of this apparent paradox (in the case of cross-fertilisation) is quite easy. A single capsule does not represent by its stigmata the individuality of the plant, i.e. its mean number of stigmata; on the other hand in seed from the same capsule fewer pollen-parents are represented than in the seed from all capsules of the plant.

(12) *Conclusions.*

(i) The character, stigmatic band number, was selected because it allowed of the crops being left pretty much to themselves, and the record of the character being formed after the crop had been harvested as a whole. Characters on the individual flower can only be observed when it is possible to isolate the individual during growth, and examine it continuously. The success of this latter method at Oxford fully justifies the time and energy given to it, but under the conditions of most of the crops here recorded full data for individual plants in the great numbers required could not be expected of those who kindly provided ground and superintended the harvesting. Further experiments are in progress which will deal more fully with other, especially colour, characters.

(ii) Local environment immensely influenced the variability and mean of the local crops. This has, however, little or no influence on the determination of heredity. What does influence heredity is differentiation in the local environment of a crop. In such a case when few parents are selected to start with a differential treatment of those individual parents may much modify results, or again a differential treatment of the sub-crops may be very detrimental to consistent results. Something of this kind seems to have affected the Kidderminster crop, for parental heredity is weakened and fraternal exaggerated—just what we should expect from the latter form of local environment differentiation.

(iii) Most plant organs being multiple in appearance, we have to apply special methods to deduce the intensity of heredity from multiple observations on the individual. It will probably be better in future experiments to confine attention to the first, or principal flower, instead of using the indirect method of homotyposis, but this will involve the observation (previous to harvesting) of individual plants in large series—500 to 1000—and much increase the labour of superintendence and observation.

(iv) Notwithstanding the difficulties referred to above we find that for a variety of plant characters in the Shirley Poppy the values of hereditary influence found are on the whole in fair agreement with the like values for man.

Undoubtedly the most self-accordant results are from Weldon's Oxford crop, where growing in pots, although it tended to produce starvelings, gave owing to the careful mixture of soil and administration of water, etc., a probably greater equality of individual environment.

Here the parental heredity was ·4 and fraternal heredity ·25, results in good accord with those for man. The Highgate results, on incomparably more meagre data, of about ·5 and ·25 respectively are also in good agreement. Enfield and Kidderminster are by no means so satisfactory. Yet when we pass from stigmatic band to colour inheritance at Enfield, we reach a good value, of ·41, for parental inheritance, and for all cases our parental means (·33 to ·34) are not below Francis Galton's first determination of heredity in man.

Hence these, the first observations on large series of the laws of inheritance in the continuously varying characters of plants, show numerical results generally in accordance with those already found for animals and insects.

The great influence of environment, and of local differences of environment, the probable stringent selection of seed and seedlings, the fewness of the original mother-plants, are all factors tending to modify and obscure the numerical results; they make the plant-problem much harder than the animal or insect problem. But the present investigation shows that there is nothing, which would lead us to suppose that the methods and results already found sufficient to describe hereditary influence in man and animals will not suffice to describe the like results in the continuously varying characters of plant life.

Further investigations on the more stable colour characters already planned will, to judge from the data here given, provide more consistent numerical results.

(v) Our experiments show that the plant, not the capsule, must be looked upon as the parent individual; closer resemblance of brother plants from seed of the same capsule being in all probability due to the restricted variety of pollen fertilising the seed of the same capsule compared with the diversity of pollen which fertilises all the seed of all capsules on the same plant.

(vi) On the completion of a long series of observations one invariably discovers much that one ought to have done and to have left undone; we reach a knowledge which *a priori* was impossible, however desirable. We have learnt to appreciate the importance of (*a*) a far greater variety in initial parentages, (*b*) the selection of characters like colour and petal form less subject than stigmata to environmental differences, and (*c*) the difficulty of avoiding differential environment when the plants are grown in ordinary garden ground. We feel that secular experiments on large series of Shirley poppies would throw much light on the laws of plant heredity, but to conduct them properly wants not only the space but the individual observation and record of each plant, which we believe can only be easily and successfully provided when the much needed Biometric Farm has been established *.

APPENDIX OF STATISTICAL TABLES.

A. Oxford Homotyposis in Stigmata.
B. Crockham " " "
C. Bookham " " "
D. Enfield I " " "
E. Enfield II " " "
F. Kidderminster " "
G. Highgate Parental Plant Mean and Offspring Capsule.
H. Oxford " " " "
I. Bookham " " " "
J. Enfield I " " " "
K. Kidderminster " " "
L. Enfield I. Parental Plant Mean and Offspring Plant Mean.
M. Oxford. Lateness of Flowering and Number of Stigmata.
N. Oxford. Parental Plant Mean and Stigmata of First Flower of Offspring.

* To any of our readers willing to assist in further observations on "first flowers," the Editors will most gladly send seed of pedigree poppies with suggestions for further work.

O. Oxford. Offspring Capsule and Parent Capsule.
P. Crockham. „ „ „
Q. Enfield I. „ „ „
R. Enfield II. „ „ „
S. Enfield II. Grandparent Plant Mean and Offspring Capsule.
T. Kidderminster „ „ „ „
U. Highgate. Fraternal Correlation First Flowers.
V. Oxford. Correlation of Capsules on Brother Plants.
W. Enfield I. „ „ „ „
X. Kidderminster. „ „ „ „
Y. Oxford. Nodal Height of Brother Plants.
Z. Oxford. Capsule Height of Brother Plants.
α. Oxford. Fraternal Correlation, Stigmata of First Flower.
β. Oxford. „ „ Number of Petals.
γ. Oxford. „ „ Petal Length.
δ. Oxford. „ „ Breadth of Margin.
ε. Oxford. „ „ Extent of Basal Patch.
ζ. Oxford. „ „ Intensity of Wrinkling.
η. Oxford. „ „ Colour of Middle Third.
θ. Enfield II. „ „ Stigmata of Capsules on Plants from same Capsule.

A. *Oxford. Homotyposis.*

Number of Stigmata on First Capsule.

Number of Stigmata on Second Capsule.

	4	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	Totals
4	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
5	1	6	3	4	2	2	1	—	—	—	—	—	—	—	—	—	19
6	—	3	35	31	35	25	17	15	9	4	—	—	2	—	—	—	176
7	—	4	31	63	73	54	43	13	12	10	2	—	—	—	—	—	305
8	1	2	35	73	116	139	130	58	63	39	18	6	—	2	—	—	682
9	—	2	25	54	139	194	242	163	101	68	33	14	2	1	—	—	1038
10	—	1	17	43	130	242	364	322	246	141	69	22	4	2	1	—	1604
11	—	—	15	13	58	163	322	483	405	294	155	46	4	3	4	1	1966
12	—	—	9	12	63	101	246	405	534	388	198	86	10	—	—	—	2052
13	—	—	4	10	39	68	141	294	388	414	328	106	9	—	1	—	1802
14	—	—	—	2	18	33	69	155	198	328	296	137	14	2	2	—	1254
15	—	—	—	—	6	14	22	46	86	106	137	66	23	5	1	—	512
16	—	—	2	—	—	2	4	4	10	9	14	23	15	10	6	3	102
17	—	—	—	—	2	1	2	3	—	—	2	5	10	6	6	3	40
18	—	—	—	—	—	—	1	4	—	1	2	1	6	6	2	2	25
19	—	—	—	—	—	—	—	1	—	—	—	—	3	3	2	—	9
Totals	2	19	176	305	682	1038	1604	1966	2052	1802	1254	512	102	40	25	9	11588

B. *Crockham. Homotyposis.*

Number of Stigmata on First Capsule.

Number of Stigmata on Second Capsule.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	Totals
6	2	4	—	4	—	2	18	34	16	4	—	—	—	—	—	—	—	—	84
7	4	2	—	9	8	8	24	49	27	9	3	1	—	—	—	—	—	—	144
8	—	—	4	18	29	46	31	40	32	8	6	4	—	—	—	—	—	—	218
9	4	9	18	70	143	239	226	270	219	67	35	25	9	1	3	—	—	3	1341
10	—	8	29	143	300	452	494	622	566	261	137	107	18	2	6	—	—	2	3147
11	2	8	46	239	452	826	985	1155	1027	391	151	78	27	2	8	—	—	1	5398
12	18	24	31	226	494	985	1414	2050	1678	745	309	190	66	15	17	—	—	4	8266
13	34	49	40	270	622	1155	2050	3714	3379	1284	474	228	69	13	12	—	—	11	13404
14	16	27	32	219	566	1027	1678	3379	3406	1727	729	377	112	26	13	—	—	10	13344
15	4	9	8	67	261	391	745	1284	1727	1476	925	485	165	44	31	—	—	1	7623
16	—	3	6	35	137	151	309	474	729	925	610	454	130	43	20	—	—	—	4026
17	—	1	4	25	107	78	190	228	377	485	454	506	207	57	41	—	—	—	2760
18	—	—	—	9	18	27	66	69	112	165	130	207	160	58	37	—	—	—	1058
19	—	—	—	1	2	2	15	13	26	44	43	57	58	20	6	—	—	—	287
20	—	—	—	3	6	8	17	12	13	31	20	41	37	6	8	—	—	—	202
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
23	—	—	—	3	2	1	4	11	10	1	—	—	—	—	—	—	—	—	32
Totals	84	144	218	1341	3147	5398	8266	13404	13344	7623	4026	2760	1058	287	202	—	—	32	61334

C. *Bookham. Homotyposis.*

Number of Stigmata on First Capsule.

Number of Stigmata on Second Capsule.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
6	—	—	—	1	1	—	1	2	1	—	—	—	—	—	—	6
7	—	—	2	1	4	3	3	1	2	1	—	—	—	—	—	17
8	—	2	8	12	12	8	13	12	17	15	3	2	—	—	—	104
9	1	1	12	30	16	20	32	28	13	27	6	—	1	—	—	187
10	1	4	12	16	44	55	42	45	24	23	3	7	1	—	—	277
11	—	3	8	20	55	40	100	55	53	42	20	20	6	4	—	426
12	1	3	13	32	42	100	134	127	97	63	28	20	2	—	—	662
13	2	1	12	28	45	55	127	126	129	85	31	22	5	—	—	668
14	1	2	17	13	24	53	97	129	144	124	47	25	6	2	—	684
15	—	1	15	27	23	42	63	85	124	146	89	37	12	6	—	670
16	—	—	3	6	3	20	28	31	47	89	78	26	8	5	—	344
17	—	—	2	—	7	20	20	22	25	37	26	24	3	7	1	204
18	—	—	—	1	1	6	2	5	6	12	8	13	2	3	—	59
19	—	—	—	—	—	4	—	—	2	6	5	7	3	2	—	29
20	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	6	17	104	187	277	426	662	668	684	670	344	204	59	29	1	4338

D. *Enfield I. Homotyposis.*

Number of Stigmata on First Capsule.

No. of Stigmata on Second Capsule.	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
9	6	—	—	2	4	3	3	—	—	—	—	—	18
10	—	4	18	24	13	9	—	—	—	—	—	—	68
11	—	18	106	151	179	96	31	10	—	1	—	—	592
12	2	24	151	368	465	485	180	47	9	—	—	—	1731
13	4	13	179	465	1026	1191	625	189	49	8	2	—	3751
14	3	9	96	485	1191	1642	1158	462	183	25	2	—	5256
15	3	—	31	180	625	1158	1140	678	309	57	8	7	4196
16	—	—	10	47	189	462	678	590	277	88	20	4	2365
17	—	—	—	9	49	183	309	277	138	55	11	7	1038
18	—	—	1	—	8	25	57	88	55	34	9	2	279
19	—	—	—	—	2	2	8	20	11	9	2	1	55
20	—	—	—	—	—	—	7	4	7	2	1	—	21
Totals	18	68	592	1731	3751	5256	4196	2365	1038	279	55	21	19370

E. *Enfield II. Homotyposis.*

Number of Stigmata on First Capsule.

No. of Stigmata on Second Capsule.	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
7	—	—	1	—	—	—	—	—	—	—	—	—	1
8	—	—	3	3	1	—	—	—	—	—	—	—	7
9	1	3	6	6	9	2	—	—	—	—	—	—	27
10	—	3	6	14	31	22	10	1	1	—	—	—	88
11	—	1	9	31	50	52	29	7	2	1	—	—	182
12	—	—	2	22	52	72	64	32	10	—	—	—	254
13	—	—	—	10	29	64	74	67	30	3	1	—	278
14	—	—	—	1	7	32	67	64	45	4	2	—	222
15	—	—	—	1	2	10	30	45	20	12	3	1	124
16	—	—	—	—	1	—	3	4	12	14	5	2	41
17	—	—	—	—	—	—	1	2	3	5	2	2	15
18	—	—	—	—	—	—	—	—	1	2	2	—	5
Totals	1	7	27	88	182	254	278	222	124	41	15	5	1244

F. *Kidderminster. Homotyposis.*

Number of Stigmata on First Capsule.

Number of Stigmata on Second Capsule.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
6	—	—	1	—	4	2	1	4	1	—	—	—	—	—	—	13
7	—	2	4	16	11	25	34	26	22	15	10	1	1	—	—	167
8	1	4	10	51	91	134	136	117	80	37	7	3	1	—	—	672
9	—	16	51	192	486	692	738	558	266	129	31	—	—	—	—	3159
10	4	11	91	486	1922	2962	3450	2884	1732	764	223	44	3	3	3	14582
11	2	25	134	692	2962	6686	8886	7958	4791	2290	687	176	12	1	—	35302
12	1	34	136	738	3450	8886	14740	15598	10350	5065	1753	409	52	13	1	61226
13	4	26	117	558	2884	7958	15598	20082	16128	8453	2945	653	141	31	4	75582
14	1	22	80	266	1732	4791	10350	16128	16722	10722	4588	1233	276	76	10	66997
15	—	15	37	129	764	2290	5065	8453	10722	9056	4869	1615	396	132	11	43554
16	—	10	7	31	223	687	1753	2945	4588	4869	3048	1288	357	123	7	19936
17	—	1	3	—	44	176	409	653	1233	1615	1288	678	248	77	8	6433
18	—	1	1	—	3	12	52	141	276	396	357	248	86	44	9	1626
19	—	—	—	—	3	1	13	31	76	132	123	77	44	28	5	533
20	—	—	—	—	3	—	1	4	10	11	7	8	9	5	—	58
Totals	13	167	672	3159	14582	35302	61226	75582	66997	43554	19936	6433	1626	533	58	329840

G. *Highgate. Parent and Offspring.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Plant. Mean Number of Stigmatic Bands.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
9·5—10·5	—	—	2	—	1	1	1	1	—	—	—	—	6
10·5—11·5	1	—	3	4	2	—	1	—	—	—	—	—	11
11·5—12·5	—	2	4	4	12	30	37	38	15	6	—	—	148
12·5—13·5	2	1	8	26	42	43	49	34	20	13	2	—	240
13·5—14·5	—	—	1	2	4	5	13	31	21	7	3	1	88
14·5—15·5	—	—	1	5	2	7	17	35	32	13	—	1	113
Totals	3	3	19	41	63	86	118	139	88	39	5	2	606

H. *Oxford. Parent and Offspring.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Plant. Mean Number of Stigmata.	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	Totals
10·099—10·599	—	1	15	14	16	22	19	14	13	2	1	1	—	—	—	—	118
10·599—11·099	—	1	1	5	8	9	17	21	16	16	9	3	—	—	—	—	106
11·099—11·599	—	—	5	11	19	35	32	43	48	36	10	3	—	—	—	—	242
11·599—12·099	5	13	28	35	53	63	94	81	85	53	31	12	4	3	2	1	563
12·099—12·599	—	10	20	40	59	97	111	122	98	55	25	6	2	—	1	—	646
12·599—13·099	—	1	6	12	23	49	87	120	127	101	72	22	6	1	—	—	627
13·099—13·599	—	—	2	4	4	9	10	17	10	20	13	6	3	1	—	—	99
13·599—14·099	—	—	1	10	5	15	22	37	50	34	12	6	2	1	—	—	195
14·099—14·599	—	—	5	12	8	10	16	14	22	14	16	12	3	—	—	—	132
Totals	5	26	83	143	195	309	408	469	469	331	189	71	20	6	3	1	2728

I. *Bookham. Parent and Offspring.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Plant. Mean Number of Stigmata.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
9·75—10·25	2	2	4	15	18	19	26	23	13	5	1	—	—	—	—	128
10·25—10·75	—	1	4	11	13	18	18	33	28	19	12	3	2	—	—	162
10·75—11·25	—	1	1	4	4	4	2	4	—	—	—	—	1	—	—	21
11·25—11·75	—	1	3	13	23	27	37	35	25	18	7	1	—	—	—	190
11·75—12·25	1	2	6	16	25	40	50	46	45	20	8	5	2	1	—	267
12·25—12·75	—	1	7	23	30	38	35	29	19	20	9	1	—	—	—	212
12·75—13·25	—	2	11	10	22	45	81	83	108	92	49	23	4	—	1	531
13·25—13·75	—	—	—	1	1	4	4	6	7	4	8	1	1	—	—	37
13·75—14·25	—	—	—	1	3	5	3	11	7	12	11	3	2	—	—	58
14·25—14·75	—	—	1	4	3	15	28	20	20	22	11	11	3	2	—	140
14·75—15·25	—	1	1	8	10	23	21	25	30	22	12	2	4	—	—	159
Totals	3	11	38	106	152	238	305	315	302	234	128	50	19	3	1	1905

J. *Enfield I. Parent and Offspring.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Plant. Mean Number of Stigmata.	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
9·75—10·25	—	—	7	20	37	71	52	28	16	5	—	—	—	236
10·25—10·75	—	—	—	—	—	—	—	—	—	—	—	—	—	0
10·75—11·25	—	—	4	8	12	37	34	19	5	2	2	—	—	123
11·25—11·75	1	3	3	26	33	75	84	61	19	11	1	—	—	317
11·75—12·25	1	4	14	68	127	186	200	111	58	15	4	—	—	788
12·25—12·75	—	1	3	13	45	64	73	43	23	5	2	—	—	272
12·75—13·25	—	3	4	34	109	219	308	274	186	74	27	6	1	1245
13·25—13·75	—	1	4	26	52	96	120	107	55	25	11	2	—	499
13·75—14·25	—	—	1	8	14	44	53	51	17	10	2	1	—	201
14·25—14·75	1	—	—	15	22	63	85	72	42	15	5	1	1	322
14·75—15·25	—	—	8	4	20	30	45	44	30	13	6	1	—	201
Totals	3	12	48	222	471	885	1054	810	451	175	60	11	2	4204

K. *Kidderminster. Parent and Offspring.*

Parent Plant. Mean Number of Stigmata on Capsule.

Offspring Plant. Number of Stigmata on Capsule.	*11·5–12·0*	*12·0–12·5*	*12·5–13·0*	*13·0–13·5*	*13·5–14·0*	*14·0–14·5*	*14·5–15·0*	*15·0–15·5*	*15·5–16·0*	*16·0–16·5*	*16·5–17·0*	*17·0–17·5*	Totals
6	—	—	—	—	—	—	—	—	—	1	—	—	1
7	—	—	1	3	—	—	1	2	—	1	1	—	9
8	—	4	1	11	5	13	3	5	2	2	3	1	50
9	4	14	14	52	15	66	11	18	10	3	3	2	212
10	22	52	52	289	63	175	62	67	41	27	27	20	897
11	62	151	118	603	151	357	179	202	82	75	54	51	2085
12	75	233	172	995	250	586	384	380	202	148	116	80	3621
13	91	294	209	1133	326	588	470	444	266	261	162	116	4360
14	60	260	184	825	339	469	409	396	297	263	129	122	3753
15	43	135	155	467	228	235	284	286	228	181	86	74	2402
16	17	55	76	214	108	89	133	130	112	99	45	53	1131
17	3	12	33	69	32	20	50	46	23	29	21	37	375
18	—	3	7	18	6	—	6	23	6	9	7	13	98
19	—	—	2	2	1	—	4	5	5	7	—	2	28
20	—	—	—	1	—	—	—	3	—	—	—	1	5
Totals	377	1213	1024	4682	1524	2598	1996	2007	1274	1106	654	572	19027

L. *Enfield I. Parent and Offspring Means.*

Offspring Plant. Mean Number of Stigmata on Capsules.

Parent Plant. Mean Number of Stigmata on Capsules.	*8·505–9·005*	*9·005–9·505*	*9·505–10·005*	*10·005–10·505*	*10·505–11·005*	*11·005–11·505*	*11·505–12·005*	*12·005–12·505*	*12·505–13·005*	*13·005–13·505*	*13·505–14·005*	*14·005–14·505*	*14·505–15·005*	*15·005–15·505*	*15·505–16·005*	*16·005–16·505*	*16·505–17·005*	*17·005–17·505*	*17·505–18·005*	Totals
10·005—10·505	—	—	—	—	1	3	5	3	10	6	8	1	4	1	2	—	—	—	—	44
10·505—11·005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
11·005—11·505	1	—	—	—	2	3	8	13	12	10	21	12	9	—	5	—	1	—	—	97
11·505—12·005	—	—	—	1	4	9	5	12	25	20	26	15	15	8	9	1	3	1	—	154
12·005—12·505	—	—	1	—	—	1	7	6	12	4	10	9	4	5	2	2	—	—	—	63
12·505—13·005	—	—	—	—	1	1	6	10	31	24	40	26	39	23	17	10	5	3	1	237
13·005—13·505	—	—	1	2	5	1	9	14	16	21	24	18	18	7	4	3	4	1	—	148
13·505—14·005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
14·005—14·505	—	—	—	—	1	2	6	3	4	5	19	6	13	5	5	4	1	—	—	74
14·505—15·005	—	—	—	—	—	—	3	—	4	7	8	5	3	3	3	1	—	—	—	37
15·005—15·505	—	—	—	—	2	1	3	1	3	4	9	8	7	4	3	2	1	1	—	49
Totals	1	—	2	3	16	21	52	62	117	101	165	100	112	56	50	23	15	6	1	903

M. *Oxford. Stigmata and Lateness of Flowering.*

Number of Stigmatic Bands on Capsule of First Flower of Plant.

	6	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	Totals
Early Flowering...	3	6	11	26	30	28	37	14	5	1	161
Late Flowering ...	1	1	7	12	21	41	28	30	18	2	161
Totals	4	7	18	38	51	69	65	44	23	3	322

N. *Oxford. Parent and Offspring's First Flower.*

Number of Stigmata on First Flower of Offspring Plant.

Mean Number of Stigmata on Capsules. Parent Plant.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	Totals
15·099—15·599	1	—	—	1	3	5	2	—	—	1	13
14·599—15·099	—	—	—	2	1	3	3	4	—	—	13
14·099—14·599	—	—	2	4	5	8	5	4	3	—	31
13·599—14·099	—	—	—	—	—	—	—	—	—	—	0
13·099—13·599	—	1	2	2	11	14	16	12	10	1	69
12·599—13·099	—	1	1	4	6	17	13	9	5	—	56
12·099—12·599	—	1	3	10	10	9	9	5	3	—	50
11·599—12·099	—	—	3	5	11	10	1	5	3	—	38
11·099—11·599	—	—	1	5	2	7	6	6	1	2	30
10·599—11·099	—	—	—	—	—	—	—	—	—	—	0
10·099—10·599	2	4	7	6	4	2	2	—	—	—	27
Totals	3	7	19	39	53	75	57	45	25	4	327

O. *Oxford Parent and Offspring Capsules.*

Offspring Capsule. Number of Stigmata.

Parent Capsule. Number of Stigmata.	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
7	—	—	—	1	4	1	5	9	4	7	1	—	—	—	—	32
8	—	1	7	8	7	19	14	3	6	1	1	—	—	—	—	68
9	—	1	5	10	16	14	17	17	7	9	—	3	—	—	—	99
10	—	2	7	2	16	19	25	14	9	3	1	—	—	—	—	98
11	3	2	7	9	12	25	13	20	17	9	5	3	—	—	—	125
12	—	3	12	18	21	15	24	16	4	4	3	1	—	—	—	121
13	—	—	7	1	19	19	20	18	15	2	4	1	—	—	—	106
14	—	2	14	16	8	20	13	12	7	4	3	2	—	1	—	102
15	—	1	3	5	4	8	15	17	21	16	3	1	1	1	—	96
16	—	1	10	11	9	17	11	14	7	7	2	—	—	—	—	89
17	—	1	1	4	14	16	9	33	21	15	2	1	—	—	—	117
18	—	—	—	9	9	8	18	18	10	6	4	1	2	1	1	87
Totals	3	14	73	94	139	181	184	191	128	83	29	14	3	3	1	1140

P. *Crockham. Parent and Offspring Capsules.*

Offspring Capsule. Number of Stigmata.

Parent Capsule. Number of Stigmata.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	Totals
7	—	—	2	7	19	26	44	70	56	18	5	3	—	—	—	—	—	—	250
8	—	1	3	7	20	34	42	50	55	25	4	1	1	—	—	—	—	—	243
9	2	2	1	10	15	26	51	72	52	26	13	9	1	—	—	—	—	—	280
10	—	2	3	8	30	40	55	72	62	72	49	26	14	2	3	—	—	—	438
11	—	—	3	6	7	22	52	52	52	34	16	9	6	1	—	—	—	—	260
12	—	—	1	2	2	3	7	11	14	15	5	2	—	—	—	—	—	—	62
13	—	—	—	4	9	22	53	41	51	40	29	24	9	4	2	—	—	—	288
14	—	—	—	4	6	6	14	29	26	14	12	5	—	1	1	—	—	1	119
15	—	2	—	6	14	36	50	84	72	35	25	11	5	2	—	—	—	—	342
16	—	—	—	1	—	9	23	33	32	36	22	6	7	1	—	—	—	—	170
17	—	—	1	5	7	26	32	36	60	52	32	10	—	—	—	—	—	—	261
18	—	—	2	8	16	16	23	31	35	42	42	39	15	7	2	—	—	—	278
Totals	2	7	16	68	145	266	446	581	567	409	254	145	58	18	8	—	—	1	2991

Q. *Enfield I. Parent and Offspring Capsules.*

Offspring Capsule. Number of Stigmata.

Parent Capsule. Number of Stigmata.	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	Totals
7	—	—	—	3	8	12	23	35	31	30	18	7	2	1	—	170
8	—	—	1	1	10	19	45	49	37	19	8	1	—	—	—	190
9	—	—	—	1	8	23	29	40	29	22	9	6	2	2	2	173
10	—	—	1	5	7	20	29	39	33	17	7	3	1	1	—	163
11	—	2	1	6	16	29	37	49	33	18	4	3	—	—	—	198
12	—	1	2	9	13	31	45	29	38	14	11	3	1	—	—	197
13	—	2	2	9	18	21	32	40	28	23	8	1	1	2	—	187
14	—	—	—	1	5	14	18	36	35	32	25	7	6	—	—	179
15	1	3	1	13	6	18	26	31	44	28	15	12	3	2	—	203
16	—	—	1	—	9	12	15	27	17	17	13	6	3	—	—	120
17	—	—	2	3	3	10	23	22	24	28	14	10	3	—	—	142
18	—	—	—	—	—	1	7	19	39	36	21	14	5	2	—	144
Totals	1	8	11	51	103	210	329	416	388	284	153	73	27	10	2	2066

R. *Enfield II. Parent and Offspring Capsules.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Capsule. Number of Stigmata.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
10	—	—	—	—	—	—	1	1	—	—	—	—	—	2
11	1	—	2	3	14	17	7	13	2	1	—	—	—	60
12	—	—	—	2	1	1	8	3	—	—	—	—	—	15
13	—	—	—	5	10	16	13	15	11	2	—	—	—	72
14	—	1	4	8	22	43	47	51	36	17	3	1	—	233
15	—	—	4	5	9	24	14	8	9	5	—	—	—	78
16	—	—	—	—	—	5	9	2	3	2	2	2	1	26
17	—	—	—	—	1	1	6	2	5	—	—	—	—	15
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	—	—	—	1	—	2	7	14	9	6	4	—	—	43
20	—	—	—	—	1	2	5	7	3	4	—	—	—	22
Totals	1	1	10	24	58	111	117	116	78	37	9	3	1	566

S. *Enfield II. Grandparent and Offspring.*

Stigmata on Capsule of Grandchild Plant.

Grandparent Plant. Stigmatic Band Mean.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
10·75—11·25	—	—	3	1	4	2	4	4	2	—	—	—	—	20
11·25—11·75	—	—	—	—	—	1	2	1	—	—	—	—	—	4
11·75—12·25	1	—	—	—	6	5	11	16	11	8	1	1	—	60
12·25—12·75	—	—	—	—	4	14	15	5	7	4	2	2	1	54
12·75—13·25	—	—	5	11	23	49	41	43	27	11	4	—	—	214
13·25—13·75	—	1	2	10	17	27	28	26	19	9	1	—	—	140
13·75—14·25	—	—	—	2	3	8	10	16	9	5	—	—	—	53
14·25—14·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·75—15·25	—	—	—	—	1	5	6	5	3	—	1	—	—	21
Totals	1	1	10	24	58	111	117	116	78	37	9	3	1	566

T. *Kidderminster. Grandparent and Offspring.*

Stigmata on Capsule of Grandchild Plant.

Grandparent Plant. Stigmatic Band Mean.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
10·005—10·505	—	—	6	16	43	107	149	170	157	113	64	19	3	—	—	847
10·505—11·005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
11·005—11·505	—	2	7	56	173	354	572	630	432	247	103	17	5	1	—	2599
11·505—12·005	—	1	4	30	130	303	516	527	423	291	172	65	16	3	1	2482
12·005—12·505	—	2	1	11	58	187	286	374	283	173	64	27	3	1	—	1470
12·505—13·005	1	2	13	45	298	605	1019	1416	1348	880	415	144	44	9	1	6240
13·005—13·505	—	2	12	54	136	330	533	548	452	308	109	38	14	4	2	2542
13 505—14·005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
14·005—14·505	—	—	8	10	36	132	265	336	356	214	123	41	8	5	1	1535
14·505—15·005	—	—	—	2	31	66	123	145	97	62	25	5	—	—	—	556
15·005—15·505	—	—	—	4	33	89	210	222	196	104	48	16	6	5	—	933
Totals	1	9	51	228	938	2173	3673	4368	3744	2392	1123	372	99	28	5	19204

U. *Highgate. Brother Plants.*

First Brother Plant. Number of Stigmata on Capsule.

Second Brother Plant. Number of Stigmata on Capsule.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
6	—	1	6	8	9	11	7	4	5	9	1	—	61
7	1	2	7	5	10	11	6	4	5	9	1	—	61
8	6	7	18	45	52	55	52	58	44	35	6	1	379
9	8	5	45	186	404	425	562	490	260	126	13	5	2529
10	9	10	52	404	120	655	990	1353	738	287	48	6	4672
11	11	11	55	425	655	874	1593	1586	695	305	29	10	6249
12	7	6	52	562	990	1593	2158	2331	1227	471	49	28	9474
13	4	4	58	490	1353	1586	2331	3230	2116	809	106	63	12150
14	5	5	44	260	738	695	1227	2116	1478	640	75	51	7334
15	9	9	35	126	287	305	471	809	640	284	31	20	3026
16	1	1	6	13	48	29	49	106	75	31	6	3	368
17	—	—	1	5	6	10	28	63	51	20	3	0	187
Totals	61	61	379	2529	4672	6249	9474	12150	7334	3026	368	187	46490

V. *Oxford. Brother Plant.*

First Brother Plant. Number of Stigmata on Capsule.

Second Brother Plant. Number of Stigmata on Capsule.	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	Totals
4	6	29	50	63	68	86	107	95	83	52	36	15	4	3	5	1	703
5	29	88	210	278	330	422	520	513	383	239	162	55	13	10	15	3	3270
6	50	210	541	774	1001	1364	1631	1567	1402	825	536	232	62	30	31	9	10265
7	63	278	774	1051	1411	2124	2560	2784	2547	1649	1023	459	124	34	33	8	16922
8	68	330	1001	1411	1900	2949	3730	3942	3622	2314	1347	541	168	55	47	16	23441
9	86	422	1364	2124	2949	4362	5680	6345	5914	3894	2236	891	261	62	46	14	36650
10	107	520	1631	2560	3730	5680	7604	8575	8515	5694	3367	1337	412	106	70	27	49935
11	95	513	1567	2784	3942	6345	8575	9789	9693	6734	4069	1641	473	96	49	16	56381
12	83	383	1402	2547	3622	5914	8515	9693	9916	7108	4065	1541	470	149	67	30	55505
13	52	239	825	1649	2314	3894	5694	6734	7108	4906	2953	1105	345	105	33	14	37970
14	36	162	536	1023	1347	2236	3367	4069	4065	2953	1844	748	225	50	14	5	22680
15	15	55	232	459	541	891	1337	1641	1541	1105	748	306	107	13	6	2	8999
16	4	13	62	124	168	261	412	473	470	345	225	107	21	4	—	—	2689
17	3	10	30	34	55	62	106	96	149	105	50	13	4	—	—	—	717
18	5	15	31	33	47	46	70	49	67	33	14	6	—	—	—	—	416
19	1	3	9	8	16	14	27	16	30	14	5	2	—	—	—	—	145
Totals	703	3270	10265	16922	23441	36650	49935	56381	55505	37970	22680	8999	2689	717	416	145	326688

W. *Enfield I. Brother Plants.*

First Brother Plant. Number of Stigmata on Capsule.

Second Brother Plant. Number of Stigmata on Capsule.	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
8	—	6	5	44	59	121	132	92	43	15	7	1	1	526
9	6	8	27	160	285	506	577	374	201	56	26	1	—	2227
10	5	27	144	430	971	1839	2036	1410	772	288	116	14	1	8053
11	44	160	430	1884	4014	7915	8671	6148	3276	1276	445	62	12	34337
12	59	285	971	4014	8636	17042	19309	13862	7958	2941	1057	158	12	76304
13	121	506	1839	7915	17042	32776	38389	28060	15712	6102	2061	301	42	150866
14	132	577	2036	8671	19309	38389	45082	34343	19427	7550	2680	386	63	178645
15	92	374	1410	6148	13862	28060	34343	26808	15581	6256	2172	393	60	135559
16	43	201	772	3276	7958	15712	19427	15581	9108	3597	1363	259	29	77326
17	15	56	288	1276	2941	6102	7550	6256	3597	1398	532	104	16	30131
18	7	26	116	445	1057	2061	2680	2172	1363	532	198	42	5	10704
19	1	1	14	62	158	301	386	393	259	104	42	10	1	1732
20	1	—	1	12	12	42	63	60	29	16	5	1	—	242
Totals	526	2227	8053	34337	76304	150866	178645	135559	77326	30131	10704	1732	242	706652

X. *Kidderminster. Brother Plants.*

Number of Stigmata on Capsule of First Brother Plant.

Number of Stigmata on Capsule of Second Brother Plant.	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Totals
6	—	—	1	1	5	8	10	34	29	24	18	5	1	—	—	136
7	—	—	4	13	98	231	343	468	364	216	73	24	5	—	—	1839
8	1	4	58	228	632	1300	2010	2192	1782	1037	429	112	31	2	—	9818
9	1	13	228	822	2709	5339	8569	9330	7333	4275	1687	553	108	17	3	40987
10	5	98	632	2709	10668	23028	37310	43163	33885	19733	8836	2735	784	153	26	183765
11	8	231	1300	5339	23028	50446	85615	98062	80399	48752	21743	6844	1742	378	61	423948
12	10	343	2010	8569	37310	85615	146104	168554	140664	85882	38603	12099	2772	734	100	729369
13	34	468	2192	9330	43163	98062	168554	198452	170465	106716	49780	16708	4250	1063	179	869416
14	29	364	1782	7333	33885	80399	140664	170465	153152	98284	46342	15892	4135	1117	210	754053
15	24	216	1037	4275	19733	48752	85882	106716	98284	64514	31968	11064	2944	881	188	476478
16	18	73	429	1687	8836	21743	38603	49780	46342	31968	15848	5820	1620	497	122	223386
17	5	24	112	553	2735	6844	12099	16708	15892	11064	5820	2320	695	200	66	75137
18	1	5	31	108	784	1742	2772	4250	4135	2944	1620	695	200	67	20	19374
19	—	—	2	17	153	378	734	1063	1117	881	497	200	67	26	7	5142
20	—	—	—	3	26	61	100	179	210	188	122	66	20	7	2	984
Totals	136	1839	9818	40987	183765	423948	729369	869416	754053	476478	223386	75137	19374	5142	984	3813832

Y. *Oxford. Brother Plants.*

First Brother Plant. Nodal Height in cms.

Second Brother Plant. Nodal Height in cms.	5·25–10·25	10·25–15·25	15·25–20·25	20·25–25·25	25·25–30·25	30·25–35·25	35·25–40·25	40·25–45·25	45·25–50·25	50·25–55·25	55·25–60·25	60·25–65·25	65·25–70·25	Totals
5·25—10·25	—	—	3	5	8	6	1	—	1	—	—	—	—	24
10·25—15·25	—	170	207	242	111	60	16	8	—	—	—	—	2	816
15·25—20·25	3	207	584	762	439	213	86	23	6	—	—	—	5	2328
20·25—25·25	5	242	762	888	626	360	186	77	28	5	—	2	11	3192
25·25—30·25	8	111	439	626	540	436	231	124	66	28	—	2	5	2616
30·25—35·25	6	60	213	360	436	468	296	218	98	58	—	8	11	2232
35·25—40·25	1	16	86	186	231	296	166	135	59	32	—	7	9	1224
40·25—45·25	—	8	23	77	124	218	135	128	46	31	—	1	1	792
45·25—50·25	1	—	6	28	66	98	59	46	18	8	—	3	3	336
50·25—55·25	—	—	—	5	28	58	32	31	8	6	—	—	—	168
55·25—60·25	—	—	—	—	—	—	—	—	—	—	—	—	—	0
60·25—65·25	—	—	—	2	2	8	7	1	3	—	—	—	1	24
65·25—70·25	—	2	5	11	5	11	9	1	3	—	—	1	—	48
Totals	24	816	2328	3192	2616	2232	1224	792	336	168	0	24	48	13800

Z. *Oxford. Brother Plants.*

First Brother Plant. Capsule Height in cms.

Second Brother Plant. Capsule Height in cms.	*18·25–23·25*	*23·25–28·25*	*28·25–33·25*	*33·25–38·25*	*38·25–43·25*	*43·25–48·25*	*48·25–53·25*	*53·25–58·25*	*58·25–63·25*	*63·25–68·25*	*68·25–73·25*	*73·25–78·25*	*78·25–83·25*	*83·25–88·25*	*88·25–93·25*	*93·25–98·25*	*98·25–103·25*	Totals
18·25— 23·25	—	—	—	—	3	2	5	4	7	1	1	—	1	—	—	—	—	24
23·25— 28·25	—	—	8	15	20	24	14	3	7	1	1	2	—	—	—	1	—	96
28·25— 33·25	—	8	24	85	88	107	72	29	28	12	1	—	—	—	—	2	—	456
33·25— 38·25	—	15	85	244	255	270	223	90	64	40	4	4	—	—	—	2	—	1296
38·25— 43·25	3	20	88	255	354	424	316	145	144	93	21	14	7	3	—	8	1	1896
43·25— 48·25	2	24	107	270	424	580	376	219	206	119	35	34	12	10	1	4	1	2424
48·25— 53·25	5	14	72	223	316	376	314	205	228	150	36	19	26	19	2	8	3	2016
53·25— 58·25	4	3	29	90	145	219	205	226	254	210	75	54	45	35	2	6	6	1608
58·25— 63·25	7	7	28	64	144	206	228	254	240	200	82	48	53	28	5	9	5	1608
63·25— 68·25	1	1	12	40	93	119	150	210	200	172	67	48	36	40	7	2	2	1200
68·25— 73·25	1	1	1	4	21	35	36	75	82	67	26	27	14	9	3	3	3	408
73·25— 78·25	—	2	—	4	14	34	19	54	48	48	27	14	8	10	2	2	2	288
78·25— 83·25	1	—	—	—	7	12	26	45	53	36	14	8	8	6	—	—	—	216
83·25— 88·25	—	—	—	—	3	10	19	35	28	40	9	10	6	6	2	—	—	168
88·25— 93·25	—	—	—	—	—	1	2	2	5	7	3	2	—	2	—	—	—	24
93·25— 98·25	—	1	2	2	8	4	8	6	9	2	3	2	—	—	—	—	1	48
98·25—103·25	—	—	—	—	1	1	3	6	5	2	3	2	—	—	—	1	—	24
Totals	24	96	456	1296	1896	2424	2016	1608	1608	1200	408	288	216	168	24	48	24	13800

a. Oxford. Brother Plants. First Flower.

First Brother Plant. Stigmata of First Flower.

Second Brother Plant. Stigmata of First Flower.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	Totals
6	2	9	15	14	17	11	10	1	2	—	81
7	9	12	30	30	26	14	13	3	4	—	141
8	15	30	44	62	58	50	52	22	9	2	344
9	14	30	62	72	101	106	103	58	29	5	580
10	17	26	58	101	120	156	159	75	50	3	765
11	11	14	50	106	156	166	207	129	67	10	916
12	10	13	52	103	159	207	174	146	70	9	943
13	1	3	22	58	75	129	146	88	52	11	585
14	2	4	9	29	50	67	70	52	36	1	320
15	—	—	2	5	3	10	9	11	1	—	41
Totals	81	141	344	580	765	916	943	585	320	41	4716

β. Oxford. Brother Plants. First Flower.

First Brother Plant. Number of Petals on First Flower.

Second Brother Plant. Petals on First Flower.	Four Petals	Four Petals and Petaloid Stamens	Five and more Petals	Totals
Four Petals	3086	118	549	3753
Four Petals and Petaloid Stamens	118	2	29	149
Five and more Petals	549	29	212	790
Totals	3753	149	790	4692

γ. *Oxford. Brother Plants.*

First Brother Plant. Length of Petal on Principal Flower in millimetres.

Second Brother Plant. Length of Petal on Principal Flower in millimetres.

	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	Totals
15	—	—	—	—	—	1	1	2	2	2	—	1	3	1	—	2	3	1	—	1	2	1	—	—	—	—	—	—	1	—	—	—	1	—	—	—	25
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	3	—	2	1	—	2	3	1	2	1	—	1	—	1	—	—	—	19
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
20	1	—	—	—	—	—	1	2	2	3	—	1	4	3	—	3	4	1	—	2	2	1	1	—	1	—	—	—	2	—	1	—	—	1	—	—	—	36
21	1	—	—	—	—	1	—	2	2	2	—	1	3	1	—	3	5	1	1	2	5	2	2	1	1	1	—	2	3	—	—	—	—	1	—	—	—	43
22	2	—	—	—	—	2	2	2	4	4	—	2	6	2	—	4	6	2	—	2	4	2	—	—	—	—	—	—	2	—	—	—	—	2	—	—	—	50
23	2	—	—	—	—	2	2	4	2	4	—	2	6	2	—	4	6	2	—	2	4	2	—	—	—	—	—	—	2	—	—	—	—	2	—	—	—	50
24	2	—	—	—	—	3	2	4	4	2	1	2	9	5	1	6	8	5	2	8	7	4	4	1	3	2	—	—	5	—	2	—	—	2	—	—	—	94
25	—	—	—	—	—	—	—	—	—	1	—	1	—	1	1	1	—	2	1	1	1	—	1	3	1	2	—	1	1	—	1	—	—	—	—	—	—	—	20
26	1	—	—	—	—	1	1	2	2	2	1	—	3	1	1	2	3	2	1	2	2	1	—	2	1	1	—	1	1	—	—	—	—	1	—	—	—	35
27	3	—	—	—	—	4	3	6	6	9	—	3	8	5	2	8	12	8	6	11	12	5	8	4	6	6	2	1	5	—	1	1	1	3	—	—	—	149
28	1	—	—	—	—	3	1	2	2	5	1	1	5	4	—	5	5	5	4	7	6	4	11	4	5	4	6	2	4	2	3	—	—	1	—	—	—	103
29	—	—	—	—	—	—	—	—	—	1	1	1	2	—	6	1	1	5	3	5	8	6	2	2	5	3	7	7	—	3	3	—	—	—	—	—	—	—	72
30	2	—	—	—	—	3	3	4	4	6	1	2	8	5	1	2	9	4	2	7	12	4	5	5	6	7	4	4	8	1	2	1	1	2	—	—	—	125
31	3	—	—	—	—	4	5	6	6	8	—	3	12	5	1	9	8	5	5	9	15	6	7	3	6	4	2	5	9	—	1	1	—	4	—	—	—	152
32	1	—	1	—	—	1	1	2	2	5	2	2	8	5	5	4	5	4	14	13	26	13	21	14	9	14	17	11	8	7	4	4	1	3	—	—	—	227
33	—	—	—	—	—	—	1	—	—	2	1	1	6	4	3	2	5	14	6	12	17	10	21	9	11	13	13	13	6	3	2	4	1	1	—	—	—	181
34	1	—	1	—	—	2	2	2	2	8	1	2	11	7	5	7	9	13	12	10	19	7	19	10	16	16	13	7	10	7	2	2	2	2	—	—	—	227
35	2	—	3	—	—	2	5	4	4	7	1	2	12	6	8	12	15	26	17	19	28	21	33	26	20	25	30	22	22	11	6	7	3	7	—	—	—	406
36	1	—	—	—	—	1	2	2	2	4	—	1	5	4	6	4	6	13	10	7	21	20	15	17	9	14	18	31	16	3	5	6	—	2	2	—	2	249
37	—	—	2	—	—	1	2	—	—	4	1	—	8	11	2	5	7	21	21	19	33	15	22	15	17	16	20	18	12	10	4	3	1	2	—	—	—	292
38	—	—	1	—	—	—	1	—	—	1	3	2	4	4	2	5	3	14	9	10	26	17	15	20	20	31	21	17	15	5	5	7	5	2	2	—	2	269
39	—	—	—	—	—	1	1	—	—	3	1	1	6	5	5	6	6	9	11	16	20	9	17	20	8	16	14	12	6	5	4	2	3	1	—	—	—	208
40	—	—	2	—	—	—	1	—	—	2	2	1	6	4	3	7	4	14	13	16	25	14	16	31	16	20	27	16	22	6	3	11	5	3	3	—	3	296
41	—	—	3	—	—	—	—	—	—	—	—	—	2	6	7	4	2	17	13	13	30	18	20	21	14	27	16	23	16	11	6	10	3	5	1	—	1	289
42	—	—	1	—	—	—	2	—	—	—	1	1	1	2	7	4	5	11	13	7	22	31	18	17	12	16	23	20	17	7	8	7	2	2	1	—	1	259
43	1	—	2	—	—	2	3	2	2	5	1	1	5	4	—	8	9	8	6	10	22	16	12	15	6	22	16	17	10	4	4	8	2	4	3	—	3	233
44	—	—	1	—	—	—	—	—	—	—	—	—	—	2	3	1	—	7	3	7	11	3	10	5	5	6	11	7	4	2	3	1	2	1	—	—	—	95
45	—	—	—	—	—	1	—	—	—	2	1	—	1	3	3	2	1	4	2	2	6	5	4	5	4	3	6	8	4	3	—	1	2	—	—	—	—	73
46	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	1	1	4	4	2	7	6	3	7	2	11	10	7	8	1	1	—	1	2	1	—	1	82
47	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	1	2	3	—	1	5	3	5	3	2	2	2	2	1	—	—	—	—	—	35
48	1	—	1	—	—	1	1	2	2	2	—	1	3	1	—	2	4	3	1	2	7	2	2	2	1	3	5	2	4	1	—	2	—	—	—	—	—	58
49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	2	—	3	1	1	3	—	—	1	—	—	—	—	1	14
50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
51	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	2	—	3	1	1	3	—	—	1	—	—	1	—	—	14
Totals	25	0	19	0	0	36	43	50	50	94	20	35	149	103	72	125	152	227	181	227	406	249	292	269	208	296	289	259	233	95	73	82	35	58	14	0	14	4480

δ. *Oxford. Brother Plants. First Flower.*

First Brother Plant. Breadth of Margin of First Flower.

Second Brother Plant. Breadth of Margin of First Flower.	Broad	Broad-Slight	Slight	Slight-None	None	Totals
Broad	122	3	108	21	411	665
Broad to Slight	3	—	4	—	6	13
Slight	108	4	86	15	381	594
Slight to None...	21	—	15	2	46	84
None	411	6	381	46	2324	3168
Totals ...	665	13	594	84	3168	4524

ε. *Oxford. Brother Plants. First Flower.*

First Brother Plant. Extent of Basal Patch of First Flower.

Second Brother Plant. Extent of Basal Patch, First Flower.	None	None-Slight	Slight	Slight-Definite	Definite	Definite-Large	Large	Totals
None	24	21	44	1	62	—	128	280
None to Slight ...	21	12	22	—	21	—	46	122
Slight	44	22	86	3	159	6	224	544
Slight to Definite	1	—	3	—	3	—	19	26
Definite	62	21	159	3	858	19	582	1704
Definite to Large	—	—	6	—	19	—	8	33
Large	128	46	224	19	582	8	1000	2007
Totals ...	280	122	544	26	1704	33	2007	4716

ζ. *Oxford. Brother Plants. First Flower.*

First Brother Plant. Intensity of Wrinkling on First Flower.

Second Brother Plant. Intensity of Wrinkling on First Flower.	Frilled	Wrinkled	Slightly-Wrinkled	Slightly-Wrinkled Not-Wrinkled	Not-Wrinkled	Totals
Frilled	10	89	75	1	22	197
Wrinkled	89	566	569	17	145	1386
Slightly-Wrinkled	75	569	718	26	275	1663
Slightly-Wrinkled to Not-Wrinkled	1	17	26	2	12	58
Not-Wrinkled	22	145	275	12	154	608
Totals	197	1386	1663	58	608	3912

η. Oxford. Brother Plants. First Flower.

First Brother Plant. Colour of Middle Third of First Flower.

Second Brother Plant. Colour of Middle Third of First Flower.	Red	Red-Pink	Pink	Pink-White	White Pink-White	White	Totals
Red	1032	55	387	154	40	11	1679
Red-Pink	55	4	49	6	1	—	115
Pink	387	49	1078	342	26	46	1928
Pink-White	154	6	342	182	10	18	712
White to Pink-White	40	1	26	10	2	6	85
White	11	—	46	18	6	2	83
Totals ...	1679	115	1928	712	85	83	4602

θ. Enfield II. Plants from seed of same Capsule.

Stigmata on Capsules of First Brother Plant.

Stigmata on Capsules of Second Brother Plant.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
6	—	—	—	—	2	1	2	4	—	—	—	—	—	9
7	—	—	—	1	—	4	8	5	7	3	1	—	—	29
8	—	—	4	10	18	37	21	23	14	4	—	—	—	131
9	—	1	10	24	38	92	68	70	50	26	6	—	—	385
10	2	—	18	38	124	270	158	137	96	37	2	1	—	883
11	1	4	37	92	270	472	358	321	259	119	26	13	5	1977
12	2	8	21	68	158	358	374	415	340	186	65	26	9	2030
13	4	5	23	70	137	321	415	492	379	228	77	16	2	2169
14	—	7	14	50	96	259	340	379	250	167	58	15	3	1638
15	—	3	4	26	37	119	186	228	167	98	27	9	1	905
16	—	1	—	6	2	26	65	77	58	27	—	—	—	262
17	—	—	—	—	1	13	26	16	15	9	—	—	—	80
18	—	—	—	—	—	5	9	2	3	1	—	—	—	20
Totals	9	29	131	385	883	1977	2030	2169	1638	905	262	80	20	10518

MISCELLANEA.

I. Note on the Results of Crossing Japanese Waltzing Mice with European Albino Races.

By A. D. DARBISHIRE.

The breeding experiments, of which this is only a preliminary account, were undertaken at Professor Weldon's suggestion with the object of throwing some light on the problem of Heredity, and especially on the Laws of Mendel.

For this purpose crosses were made between the Japanese "waltzing" mouse and the common albino mouse. These animals were used because they have already been made the subject of similar experiments, the results of which have been recorded first by Haacke*, who gives no numerical statements, but says that when a Japanese mouse is crossed with an albino the offspring is wild-coloured or black, sometimes with a white spot on the forehead or belly, and secondly by von Guaita† who gives elaborate tables, showing that the first cross-bred generation has always the colour of wild mice, while individuals of subsequent generations are white, black, brown or piebald. The cross-bred forms of the first generation did not exhibit the "dancing" movements of the Japanese mice, while a certain proportion of individuals in subsequent generations did so. Bateson‡ has suggested that the results obtained by these observers, and by others, "show an essential harmony in the fact that both found *albino* an obvious recessive, pure almost without exception, while the coloured forms show various phenomena of dominance." Lack of uniformity in the characters of the first cross-bred generation is elsewhere attributed by Bateson to impurity in the stock used§.

The experiments were made, then, in order to answer two simple questions:

(1) Does the first generation of cross-bred individuals, produced by pairing albino and Japanese waltzing mice, exhibit such a uniformity of colour as will justify us in considering that Mendel's Law of Dominance applies to it?

(2) Does the Mendelian Law of Segregation apply to the offspring of hybrids resulting from such crosses?

* "Über Wesen, Ursachen und Vererbung von Albinismus, etc.," *Biologisches Centralblatt*, xv. 1895, pp. 44 *et seq.*

† "Versuche mit Kreuzungen von verschiedenen Rassen des Hausmaus," *Berichte d. naturforsch. Gesellsch. Freiburg*, x. (1898) pp. 317–332, and xi. (1900) pp. 131–138.

‡ *Mendel's Principles of Heredity*, Cambridge, 1902, p. 174.

§ *Report to the Evolution Committee of the Royal Society*, 1902, p. 145.

The answer to the first question has been obtained, and it is in the negative: the answer to the second has not, as the hybrids are not yet old enough to be paired. The experiments are as yet only beginning; but the evidence concerning the characters of the first generation is already greater than that obtained by von Guaita, who only observed the result of crossing four pairs of mice, while the litters already obtained by me are the result of nine crosses.

The Parent forms.

(*a*) *The Japanese Waltzing Mice.*

The average size of these mice is slightly smaller than that of the common house mouse: they are characterized by their faculty of spinning round, which is due to abnormality of the semicircular canals; and by the restlessness of their demeanour when not dancing. The ground colour of their coat is pure white; but there is always a variable amount of pale fawn on the cheeks, shoulders and rump; the arrangement of the colour on the mouse is seen in Fig. 1. They have pink eyes.

Our stock of waltzing mice arrived in December last: from that time till August they bred freely; and in all cases the offspring of two waltzing mice were indistinguishable from their parents, except for slight differences in the distribution of the fawn colour on the body: that is to say the original stock are shown by the character of their offspring to have been pure-bred. This fact is emphasized in order to remove any suspicion which may arise in the mind of some careful critic that the waltzing mice dealt with may have been dominant hybrids.

(*b*) *The Albino Mice.*

The mice used were the true albinos with pink eyes which are familiar to everyone: they may be roughly divided into two categories:—

(i) Pure-bred Albinos from the well-known mouse breeder Mr Steer and others.

(ii) Cross-bred Albinos which have appeared from time to time in the litters of piebald mice kept in the Oxford Laboratory for embryological purposes.

The cross-bred albinos were used advisedly: for albinism is said to be a recessive character; and this being so any albino is perfectly pure regardless of ancestry: that is to say, on the Mendelian hypothesis it makes no difference, as far as its offspring is concerned, whether a certain albino is the child of piebald parents or whether its parents have been pure white for many generations.

On the Mendelian hypothesis the ancestry of the albinos should make no difference: we shall see that, as a matter of fact, it probably does.

The Hybrids.

The number of hybrid families in which the colours can be seen is at present nine. The coloration of the hybrid differs from that of either parent in the fact that with two exceptions there are patches of colour which it is difficult to distinguish from that of the common house mouse.

The hybrids can be roughly classified according to the distribution of this colour on their bodies.

(*a*) Mice in which the distribution of the gray colour corresponds roughly with the distribution of fawn colour in the waltzing mouse. Fig. 2.

(*b*) Mice in which the gray colour covers much more of the body. Fig. 3.

(*c*) Mice which are all gray except on the belly and tail which are always nearly white. At a casual glance this type might easily be mistaken for a house mouse; but inspection of its belly reveals its hybrid nature. Fig. 4.

(*d*) Mice which are all fawn coloured except on the belly. The colour is like that of the markings on the Japanese waltzers: its distribution corresponds to that of the gray in (*c*).

a and *b* merge into one another, and the standard of separation is more or less arbitrary; but the line dividing *b* or *a* from *c* is at present quite sharp.

The following is a record of the crosses:

Cross 1. ♀ cross-bred albino.
♂ Japanese waltzer.
Young. 4 *a*.
2 *b*.

Cross 6. ♀ cross-bred albino.
♂ Japanese waltzer.
Young. 1 *a*.

Cross 7. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 3 *a*.
2 *b*.
3 *c*.

Cross 8. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 5 *c*.

Cross 9. ♀ cross-bred albino.
♂ Japanese waltzer.
Young. 6 *a*.

Cross 12. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 4 *c*.
2 *d*.

Cross 13. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 2 *b*.
2 *c*.

Cross 16. ♀ cross-bred albino.
♂ Japanese waltzer.
Young. 6 *b*.

Cross 20. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 4 *a*.
1 *b*.
1 *c*.

It will be seen by reference to the above figures that there are 18 *a*, 13 *b*, 15 *c* and 2 *d*, that is to say, 31 cases at least out of 48 in which albinism is not recessive: and even in *c*, which is a gray mouse superficially not unlike the house mouse, we by no means find a complete disappearance of whiteness; for as it has already been said, their bellies are nearly white; which is not true of the house mouse: and the bellies of *d* are like those of *c*.

The Influence of Ancestry.

The above tables show that hybrids with the colour of the house mouse all over (except the belly) appear only in the litters of pure-bred albinos, *i.e.* 3 *c* in Cross 7, and 5 *c* in Cross 8, 4 *c* in Cross 12, 2 *c* in Cross 13, and 1 *c* in Cross 20, whereas they do not appear in the litters of cross-bred mothers. Of course so small a number of trials does not prove a definite rule: but the result observed may be connected with the fact that von Guaita who used albinos in-bred for 29 generations always got hybrids identical with the house mouse. And it looks as if it could be said that the more in-bred an albino is the less power it has of transmitting its whiteness. But many more crosses must be made before any definite statement can be made on this point.

Fig. 1.

Fig. 2.

Fig. 3.

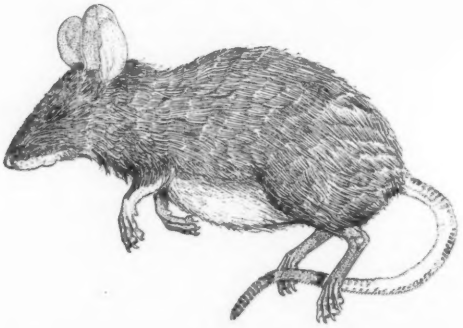

Fig. 4.

II. Interpolation by Finite Differences. (*Two Independent Variables.*)

By W. PALIN ELDERTON.

Let it be required to find $u_{p,r}$ in terms of $u_{0,0}, u_{0,1}, \dots u_{1,0} \dots u_{-1,0} \dots, u_{0,-1} \dots$, where p and r are both <1.

$$u_{p,r}=(1+\Delta_1)^p u_{0,r}=(1+\Delta_1)^p (1+\Delta_2)^r u_{0,0}$$
$$=\left\{1+(p\Delta_1+r\Delta_2)+\frac{1}{2!}(p^{(2)}\Delta_1^2+2pr\Delta_1\Delta_2+r^{(2)}\Delta_2^2)\right.$$
$$\left.+\frac{1}{3!}(p^{(3)}\Delta_1^3+3p^{(2)}r\Delta_1^2\Delta_2+3pr^{(2)}\Delta_1\Delta_2^2+r^{(3)}\Delta_2^3)\dots\right\} u_{0,0} \quad \dots\dots\dots(1),$$

where $p^{(n)}=p(p-1)\dots(p+n-1)$ and Δ_1 and Δ_2 represent operations with respect to p and r respectively.

If we use the expression $\{1+(p\Delta_1+r\Delta_2)\}u_{0,0}$ for $u_{p,r}$ we require only 3 values of the function, while if we take in the next term in round brackets we require 6 values, but the objection to the formulæ seems to be that the values of the function which we use are not necessarily the nearest values to $u_{p,r}$. For practical purposes it would be better to have the expression in terms of the function rather than in terms of the differences, as the calculation of differences running in three directions is troublesome and the work is likely to contain slips*.

The following scheme shews the form of the problem and gives an idea of which are the best functions to use for interpolation :

$u_{-1,-1}$	$u_{-1,0}$	$u_{-1,1}$	$u_{-1,2}$
$u_{0,-1}$	$u_{0,0}$	$u_{0,1}$	$u_{0,2}$
	$u_{p,r}$		
$u_{1,-1}$	$u_{1,0}$	$u_{1,1}$	$u_{1,2}$
$u_{2,-1}$	$u_{2,0}$	$u_{2,1}$	$u_{2,2}$.

Now $u_{p,0}=u_{0,0}+p(u_{1,0}-u_{0,0})$, $u_{p,1}=u_{0,1}+p(u_{1,1}-u_{0,1})$

and interpolating between these values we get (when $s=1-r$ and $q=1-p$)

$$u_{p,r}=qsu_{0,0}+qru_{0,1}+psu_{1,0}+pru_{1,1}+ \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots(2).$$

In most cases occurring in practice the coefficients in (2) can be calculated at sight and the labour of the whole interpolation is very small. The coefficients can easily be remembered by considering the distances of the required function from a given value and bearing in mind that the nearer the position of the given value to that of the required value the larger its coefficient.

For some purposes (2) will not be sufficiently accurate and we must seek for a similar formula involving more terms.

For this purpose Lagrange's Interpolation formula can be used and would be the only one applicable when the intervals between the given values of the function are not all equal. We should first find $u_{p,a}$, $u_{p,b}\dots$ and then $u_{p,r}$ by independent interpolations or else by working out coefficients as in formula (2). If we consider Lagrange's formula, viz.

$$u_p=\frac{(p-b)(p-c)\dots}{(a-b)(a-c)\dots}u_a+\frac{(p-a)(p-c)\dots}{(b-a)(b-c)\dots}u_b+\dots$$

[* The biometrician has repeatedly to use tables of double entry: e.g. in the cases of skew variation when using the G-integral (*Brit. Assoc. Report*, 1899), in dealing with goodness of fit (*Biometrika*, Vol. I. p. 155), in finding the influence of natural selection on correlation (*Phil. Trans.* A, Vol. 200, p. 64), etc., etc. Hence the importance of a good method of interpolation. Ed.]

† See note by T. G. Ackland, *Journal Institute of Actuaries*, Vol. 32, p. 286.

and put $a=0$, $b=1$, $c=-1$, $d=2$, $e=-2$, etc., we have

$$u_p=\frac{(p-1)(p+1)(p-2)(p+2)\ldots}{(-1)(1)(-2)(2)\ldots}u_0+\frac{p(p+1)(p-2)(p+2)\ldots}{(1)(2)(-1)(3)(-2)\ldots}u_1+\frac{p(p-1)(p-2)(p+2)\ldots}{(-1)(-2)(1)(-3)(2)\ldots}u_{-1}\text{ etc.}$$

$$=p_{2n}\left\{c_0\frac{u_0}{p}+c_1\frac{u_1}{p-1}+c_{-1}\frac{u_{-1}}{p+1}+\ldots+\frac{c_t u_t}{p-t}+\ldots\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3),$$

where it is assumed that $2n$ terms are used, so that

$$p_{2n}=(p+n-1)\ldots(p-n) \text{ and } c_t=(-1)^{n-t}\frac{1}{\lfloor n-t \rfloor \lfloor n+t-1}.$$

These two functions, p_{2n} and c_t, can be calculated easily as in the case of a two-variables interpolation n will not be greater than 2 or for a one-variable than 4. Taking $n=4$, $t=2$ as an example

$$c=\frac{1}{\lfloor 2 \rfloor 5}=\frac{1}{240}.$$

For the two-variables we shall use (3) for obtaining an interpolated value in the following form

and

$$\left.\begin{aligned} u_{p,r}&=p_{2n}\left\{c_0\frac{u_{0,r}}{p}+c_1\frac{u_{1,r}}{p-1}+\ldots+\frac{c_t u_{tr}}{p-t}+\ldots\right\}\\ u_{p,r}&=r_{2n}\left\{c_0\frac{u_{p,0}}{r}+c_1\frac{u_{p,1}}{r-1}+\ldots+\frac{c_t u_{pt}}{r-t}+\ldots\right\}\end{aligned}\right\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

By this method we see the connection between Professor Everett's Central Difference* and Lagrange's formulæ, for if we write the differences in the former in terms of the function and take p_{2n} outside a bracket we see that the two are alike. In the actual work in the case of a two-variables interpolation $p_{2n}\times r_{2n}$ could easily be obtained and then we could find the coefficients $\div p_{2n}\times r_{2n}$. Such reduced coefficient for $u_{t,s}$ would be

$$\frac{c_t\,c_s}{(p-t)\,(r-s)}.$$

Let us find as an example of the formulæ an interpolated value for ages 51 and 28 from the following values at 3 °/₀ interest from the H^M Joint Life Annuity Table†.

Age of Younger Life	Age of Elder Life			
	45	*50*	*55*	*60*
20	14·1936	12·8092	11·2791	9·6503
25	14·0130	12·6787	11·1886	9·5902
30	13·7313	12·4690	11·0378	9·4855
35	13·3625	12·1954	10·8436	9·3536

* $u_p=qu_0+\frac{(q+1)\,q\,(q-1)}{\lfloor 3}a_2+\frac{(q+2)\ldots(q-2)}{\lfloor 5}a_4+\ldots+pu_1+\frac{(p+1)\,p\,(p-1)}{\lfloor 3}b_2+\frac{(p+2)\ldots(p-2)}{\lfloor 5}b_4+\ldots$

where a_2, $a_4\ldots$ are even central differences of u_0 and b_2, $b_4\ldots$ those of u_1.—See *Journal Institute of Actuaries*, Vol. 35, p. 452.

† H^M is the name given to the mortality table constructed by the Institute of Actuaries (1869) from the experience of healthy male lives assured by English offices.

We obtain the following values

By (1) 1st Differences (3 values).....12·2549
(1) 2nd „ (6 values).....12·2773
(2) 4 values ....12·2619
(4) 16 values .....12·2815
The value given in the Tables is .....12·2811.

In applying formulæ (4) if we use $2n$ terms in each equation we require $4n^2$ terms altogether —thus 4, 16, 36... are the numbers of terms that will occur. Of course if we use an odd number of terms, say $2n+1$, the formula reads

$$u_{p,r}=p_{2n+1}\left\{\frac{c_0u_0}{p}+\ldots+\frac{c_tu_t}{p-t}+\ldots\right\} \text{ and } u_{p,r}=r_{2n+1}\left\{\frac{c_0u_0}{r}+\ldots+\frac{c_tu_t}{r-t} \text{ etc.}\right\}$$

where $p_{2n+1}=(p+n)(p+n-1)\ldots(p+1)p(p-1)\ldots(p-n)$. The formulæ can also be used when m terms are used in one direction and n in the other.

As an example of the full working take the table of joint life annuities given and find the annuity on two lives aged 51·27355 and 28·63984. Here $p=\cdot25471$, $r=\cdot72797$.

$\log(p+1)$	·0985433
$\log(2-r)$	·1044973
$\log(2-p)$	·2418676
$\log(r+1)$	·2375362
$\log(p)$	$\bar{1}$·4060460
$\log(1-r)$	$\bar{1}$·4346168
$\log(1-p)$	$\bar{1}$·8723253
$\log(r)$	$\bar{1}$·8621135
$\log p_4r_4$	$\bar{1}$·2575460

$c_0=\frac{1}{2}$, $c_1=-\frac{1}{2}$, $c_2=\frac{1}{6}$, $c_{-1}=-\frac{1}{6}$,
$\log\frac{1}{2}=\bar{1}\cdot6989700$ and $\log\frac{1}{6}=\bar{1}\cdot2218487$,

and we get the following expressions corresponding to {...} in (4), where [·2...] represents antilog ·2....

"p" terms $-[\bar{1}\cdot1233054]u_{-1}+[\ \cdot2929240]u_0+[\bar{1}\cdot8266447]u_1-[\bar{2}\cdot9799811]u_2$,
"r" terms $-[\bar{2}\cdot9843125]u_{-1}+[\bar{1}\cdot8368565]u_0+[\ \cdot2643532]u_1-[\bar{1}\cdot1173514]u_2$,

(+) 1·1520924	(−) 1·1075220	(−) 1·0512745	(+) ·9845408
$\bar{2}$·1076179	$\bar{1}$·2772365	$\bar{2}$·8109572	$\bar{3}$·9642936
(−) 1·1465311	(+) 1·1030747	(+) 1·0487757	(−) ·9818277
$\bar{2}$·9601619	·1297805	$\bar{1}$·6635012	$\bar{2}$·8168376
(−) 1·1377117	(+) 1·0958316	(+) 1·0428827	(−) ·9770602
$\bar{1}$·3876586	·5572772	·0909979	$\bar{1}$·2443343
(+) 1·1258878	(−) 1·0861960	(−) 1·0351735	(+) ·9709788
$\bar{2}$·2406568	$\bar{1}$·4102754	$\bar{2}$·9439961	$\bar{2}$·0973325

The upper line in each case gives the logarithm of the joint life annuity, and the lower line the sum of the logarithms of the coefficients shewn in the two expressions marked "p" terms

and "r" terms. The two lines are added at sight and the anti-logarithm found. The sign of each expression, given in brackets, is a great help to rapid work.

+	−
·18185	1·27847
·23257	3·35251
17·09435	2·42526
44·98925	3·13669
5·15557	·72984
13·61071	·95317
·08889	·62902
·11703	1·66492
81·47022	14·16988
14·16988	
log 67·30034 =	1·8280173
	$\bar{1}$·2575460
	1·0855633 ... 12·17765

Using the annuities for 51, 28 ; 51, 29 ; 52, 28 and 52, 29 with formula (2) I get 12·1775.

III. Variation in the Moscatel (*Adoxa Moschatellina*, L.).

By HENRY WHITEHEAD.

The mode of arrangement of flowers in globose heads in the Moscatel (*Adoxa moschatellina*, L.) is often cited as an example of packing the maximum number of flowers in the minimum of space. An inflorescence of the Moscatel generally consists of five flowers—one terminal and four lateral. The terminal flower has two bracts, four petals, four branched stamens and four carpels ; and each of the lateral flowers possesses three bracts, five petals, five branched stamens and five carpels. This is the most common form of arrangement, but the plant varies considerably and the following figures show that only 55 per cent. of the numbers counted agree with the above description, and 51 other methods of arrangement are noted.

The number of flowers in the inflorescence is a variable quantity as well as the number of parts in the perianth, and for the sake of convenience these variations are considered separately.

The specimens examined were obtained from three localities, Chislehurst in Kent, Caterham in Surrey, and Theydon Garnon in Essex.

I. *Variation in the number of flowers in an inflorescence.*

The number of flowers constituting an inflorescence varies from three to ten. 1071 inflorescences were counted and the following results were obtained :

TABLE I.

Number of flowers	3*	4	5	6	7	8	9	10
Actual quantities...	6	71	934	26	24	7	1	2
Percentages	0·6	6·6	87·2	2·4	2·1	0·6	0·1	0·2

Diagram I. represents the above figures in a graphical form.

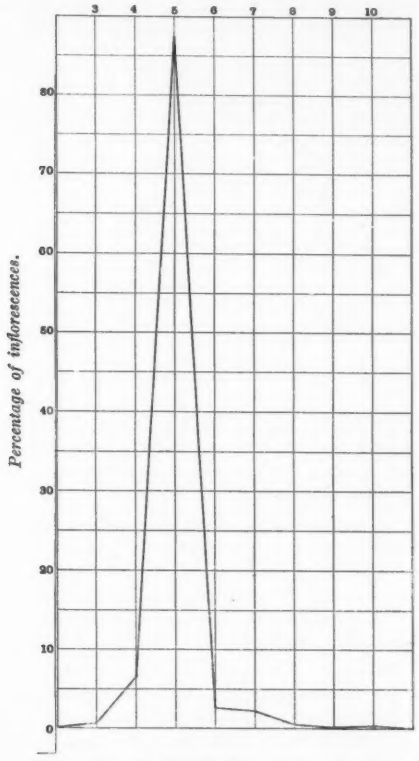

DIAGRAM I. Number of flowers per inflorescence.

In Gerard's *Herbal* (2nd Ed. 1633), p. 1090, there is a note relating to the Moscatel which is interesting. He says "the floures grow clustering on the top of a stalk, commonly five or seven together,.....; it floures in Aprill and is to be found in divers places amongst bushes at that time, as in Kent about Chiselhurst, especially in Pits his wood and at the further end of Cray heath on the left hand under a hedge among bryers and brambles, which is his proper seat." Gerard gives an illustration of the Moscatel which shows two flowering stems, one

of these has a clustered group at the top with one small lateral flower some distance down the stem, and the other a clustered group with two lateral flowers in a lower position. Several plants bearing seven flowers were found near the spot mentioned by Gerard as "Cray heath." It seems strange that Gerard should give the numbers as five to seven when the four-flowered inflorescences occur more than twice as often as either the six-flowered or the seven-flowered.

II. *Variation in the number of petals.*

Before considering the variation in the number of petals it would be as well to say a few words about the mode of arrangement of the parts of a flower of the normal type.

The tips of each of the uppermost petals of the four pentamerous lateral flowers (1, Diagram II) fit into the space between two petals (4) of the terminal and tetramerous flower.

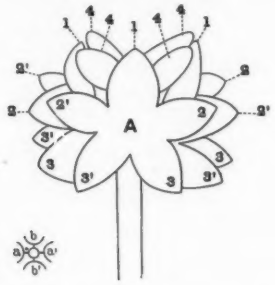

DIAGRAM II. Plan and side view of inflorescence without terminal flower.

The arrangement of the remaining petals is shown in Diagram II, where the petal indicated by 2′ in the expanded lateral flower shown in the diagram touches petal 2 of the flower on the left hand, while petal 2 of flower *A* touches 2′ of its neighbour on the right. The petals marked 3 and 3′ touch in a similar manner.

The order in which the various flowers and petals in an inflorescence generally open is very curious. First, the four petals of the terminal flower expand simultaneously, then two lateral buds which are diametrically opposite (*a* and *a*′) open, and finally the opening of the other pair of buds (*b* and *b*′) takes place. The order in which the petals of the pentamerous flowers expand is 1, 2 and 2′, and 3 and 3′. Mathematical precision is necessary where the flowers are packed in such a neat fashion, otherwise the struggle for space would result in a haphazard arrangement and consequently a loss of space.

The terminal flower is the least variable of all. Out of 763 terminal flowers 754 (about 99 per cent.) were found to be tetramerous; 4 being pentamerous and 5 trimerous. The fixity in the number of parts of the terminal flower is used for the purpose of classifying and tabulating the variations from the normal type as given in Table II.

The numbers given in Table II. lines 16—20 deserve special notice. Line 16 shows that 420 inflorescences with 4 pentamerous lateral flowers (represented by the entry 4 in column 5) were noted. In line 17 where there are 3 "fives" and 1 "four" in the inflorescence the number has dropped to 125. In line 18 where there are 2 "fives" and 2 "fours" the number is 59, and lines 19 and 20 show a further decrease in the totals as the florets with four petals take the places of those with five. The same rule holds in cases where the numbers of flowers in the inflorescences are seven, six and four.

TABLE II.

Variations in the Number of Divisions of the Corolla.

Terminal Flower	Total Number of Flowers per inflorescence	Distribution of Petals in lateral flowers*						Locality			Totals	Number of Row
		8	*7*	*6*	*5*	*4*	*3*	Near Chislehurst, Kent	Caterham, Surrey	Theydon Garnon, Essex		
Trimerous	4	—	1	—	2	—	—	1	—	—	1	1
	4	—	—	—	2	1	—	1	—	—	1	2
	4	—	—	—	1	2	—	—	—	1	1	3
	4	—	—	—	—	3	—	—	—	2	2	4
Tetramerous	3	1	—	—	1	—	—	1	—	—	1	5
	3	—	—	—	2	—	—	4	—	—	4	6
	4	—	—	—	3	—	—	16	3	2	21	7
	4	—	—	—	2	1	—	4	1	—	5	8
	4	—	—	—	1	2	—	3	—	1	4	9
	4	—	—	—	—	3	—	—	—	1	1	10
	4	—	—	2	—	1	—	1	—	—	1	11
	4	—	—	1	2	—	—	2	—	—	2	12
	4	—	—	1	1	1	—	1	—	—	1	13
	4	—	—	1	—	2	—	1	—	—	1	14
	4	—	—	—	2	—	1	1	—	—	1	15
	5	—	—	—	4	—	—	283	30	107	420	16
	5	—	—	—	3	1	—	91	6	28	125	17
	5	—	—	—	2	2	—	38	10	11	59	18
	5	—	—	—	1	3	—	27	1	4	32	19
	5	—	—	—	—	4	—	10	2	2	14	20
	5	—	—	3	—	1	—	—	1	—	1	21
	5	—	—	2	2	—	—	1	1	—	2	22
	5	—	—	2	1	1	—	1	—	—	1	23
	5	—	—	1	3	—	—	4	3	—	7	24
	5	—	—	1	2	1	—	3	1	1	5	25
	5	—	—	1	1	2	—	1	1	—	2	26
	5	—	—	—	3	—	1	1	—	—	1	27
	5	—	—	—	2	1	1	1	—	—	1	28
	5	—	—	—	1	2	1	7	—	—	7	29
	5	—	—	—	1	1	2	—	—	1	1	30
	5	—	—	—	—	3	1	2	—	—	2	31
	6	—	—	1	4	—	—	—	1	—	1	32
	6	—	—	—	5	—	—	5	—	1	6	33
	6	—	—	—	4	1	—	1	—	—	1	34
	6	—	—	—	3	2	—	1	—	—	1	35
	6	—	—	—	2	3	—	1	—	—	1	36
	6	—	—	—	2	2	1	—	2	—	2	37
	6	—	—	—	1	4	—	2	2	—	4	38
	7	—	—	—	6	—	—	5	—	—	5	39
	7	—	—	—	5	1	—	1	1	1	3	40
	7	—	—	—	4	2	—	—	1	—	1	41
	7	—	—	—	4	1	1	1	—	—	1	42
	7	—	—	—	3	3	—	—	1	—	1	43
	7	—	—	—	3	2	1	1	—	—	1	44
	7	—	—	—	—	6	—	1	—	—	1	45
	7	—	—	—	—	3	3	2	—	—	2	46
	8	—	—	—	6	1	—	1	—	—	1	47
Pentamerous	5	—	—	—	4	—	—	1	—	—	1	48
	5	—	—	—	2	2	—	1	—	—	1	49
	5	—	—	—	1	2	1	1	—	—	1	50
	6	—	—	—	2	3	—	—	1	—	1	51

* The figures at the top of this column denote the number of petals per flower. The figures in the vertical columns denote the number of flowers of each type.

It will be seen that the corollas of the *Adoxa* are 8-, 7-, 6-, 5-, 4- and 3-partite, and the actual number of each of these types of corolla is shown in Table III, while Diagram III gives the results in a graphical form.

TABLE III.

	3	4	5	6	7	8
Number of flowers not including terminal flowers	25	514	2494	30	1	1
Number of terminal flowers	5	754	4	—	—	—
Totals	30	1268	2498	30	1	1
Percentages of totals (to nearest 10th) ...	0·8	33·1	65·3	0·8	—	—
Assuming that all inflorescences were normal then percentages of totals would be	—	20	80	—	—	—

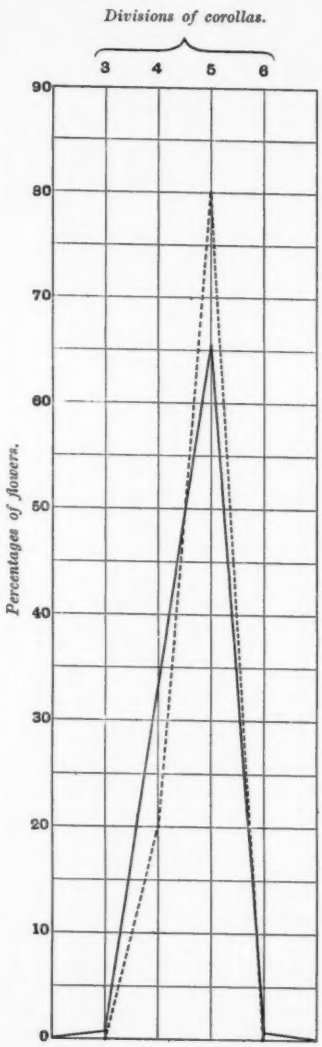

Diagram III.

——— Curve plotted from actual numbers.
- - - - - Curve if all the inflorescences were normal.

The statistics presented in this paper show that the inflorescences of the Moscatel vary considerably from the normal type. A normal inflorescence consisting of five flowers has one tetramerous and four pentamerous, that is, a percentage ratio of 20 to 80, but the ratio of the observed percentages is as 33·1 to 65·3. The variation from the normal type therefore favours the tetramerous type more than the pentamerous type, while sports occur in the form of 3-, 6-, 7-, and 8-partite corollas though these form less than 2 per cent. of the total.

NOTE.

At the suggestion of Prof. Karl Pearson several old herbaria at Kew, the British Museum and at Cambridge were examined in order to see whether any of the specimens agree with the figure given in Gerard's *Herbal*, 2nd Ed., p. 1091 (edited by Thos. Johnson). None were found exhibiting the peculiar racemose arrangement which is so marked in his drawing. The first edition (published 1597) has an entirely different figure of *Adoxa*, which has evidently been borrowed from the *Kräuter Buch* of Tabernæmontanus. In this case all the lateral flowers are tetramerous.

I am indebted to Miss K. M. Hall, Curator of the Stepney Borough Museum, for much help and many useful suggestions.

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IV. Seasonal Change in the Characters of Aster prenanthoides, Muhl.

In the last number of *Biometrika* (pp. 304—315) a collection of evidence was published, showing the existence of periodic changes in the mean and modal numbers of floral organs, and in the constants expressing the variation of these numbers, in several plants. Just before the publication of this article, Mr G. H. Shull published in the *American Naturalist*, Vol. XXXVI. No. 422, an elaborate "Quantitative Study of Variation in the Bracts, Rays and Disk-florets "of *Aster Shortii* Hook., *A. Novae Angliae* L., *A. puniceus* L., and *A. prenanthoides* Muhl., "from Yellow Springs, Ohio." In this paper the constants expressing the variation in and correlation between bracts, rays, and disk-florets are fully determined for each species; and in addition, the author describes the results obtained by examining four sets of flowers, collected from the same series of individuals of *A. prenanthoides* on four different days. The first of these four gatherings was made on September 27, the last on October 8; a comparison between

the four sets of results gives a remarkable and instructive demonstration of the change which may occur in individual plant characters within a short space of time. The following table, abridged from the fuller tables given by Mr Shull, shows some features of the change.

TABLE I.

Variation in Number and S. D. of Floral Organs on four different Days.

	September 27 (117 Capitula)	September 30 (143 Capitula)	October 4 (139 Capitula)	October 8 (176 Capitula)
Mean No. Bracts ...	47·410 ± 0·345	44·343 ± 0·291	43·835 ± 0·302	41·920 ± 0·249
S. D. of Bracts ...	5·524 ± 0·244	5·152 ± 0·205	5·276 ± 0·213	4·890 ± 0·176
Mean No. Rays ...	30·769 ± 0·249	28·706 ± 0·201	28·252 ± 0·200	26·335 ± 0·153
S. D. of Rays ...	3·986 ± 0·176	3·569 ± 0·142	3·501 ± 0·142	3·010 ± 0·108
Mean No. Disk-florets	56·427 ± 0·249	51·713 ± 0·282	49·158 ± 0·279	45·778 ± 0·242
S. D. Disk-florets ...	3·986 ± 0·176	4·995 ± 0·199	4·885 ± 0·198	4·777 ± 0·171

The change in mean and standard deviation of all the organs studied during the short interval between September 27 and September 30 is of especial interest in connection with the asserted multimodal distributions so often described in floral organs.

Mr Shull lays little stress on the peaks of his frequency curves, but it is worth notice that in this carefully collected material there is not a single case of a many-peaked curve in which the "modes" coincide with the numbers of the Fibonacci series, and the author criticises the process by which previously recorded "modes" are brought into accord with this series. He says "The members of the series, along with Ludwig's 'Unterzahlen,' which are "made up from the Fibonacci numbers by multiplication or addition,—e.g., $10=(2\times5)$, "$29=(8+21)$ etc.,—include so large a proportion of all the smaller numbers that many modes "must fall on or near one of them, even if there be no fundamental relation existing between "this complex series and the number of floral parts or other organs under consideration. "To account for modes which do not fall on any of these, Ludwig creates the 'Scheingipfel,' "which is formed by the overlapping of curves having their modes on adjacent numbers of "the Fibonacci-Ludwig complex. Thus, if the maximum fall on 9, it is a 'Scheingipfel' "formed by the union of curves having maxima upon 8 and 10; if it fall upon 11, it is "made up of curves having maxima upon 10 and 13, etc. It is evident that such a scheme "will furnish an explanation of almost any condition which might arise."

It is to be hoped that Mr Shull's admirable essay will induce future investigators to consider more carefully the sources of error in their process of collection before they assert that even statistically significant "multimodality" is an indication of actual polymorphism in plants, and to realise the importance of prolonged study before the differences between local races can be usefully insisted upon.

W. F. R. W.

ON THE NUMBER AND ARRANGEMENT OF THE BONY PLATES OF THE YOUNG JOHN DORY.

By L. W. BYRNE.

THE accompanying notes were made some years ago at the suggestion of my friend Mr E. W. L. Holt*. It had been my intention to make them more complete by the examination of adult specimens from the same locality, but for this I have had neither time nor material and they are only published now as throwing, perhaps, a little light upon one of the natural causes which may have assisted in the derivation of the asymmetrical *Pleuronectidae* from some more normally-formed member of the group *Zeorhombi*†.

The material examined consisted of 250 specimens of *Zeus faber* from $2\frac{3}{4}$ to 5 inches in length and probably about a year old, captured by Plymouth trawlers.

For convenience the bony plates are in these notes expressed by four letters or numbers as follows:—

$$\text{Left } \frac{\text{Dorsal}}{\text{Ventral}} \text{ Right} = \text{Left } \frac{9 \mid 7}{8 \mid 10} \text{ Right}$$

the diagram $\frac{9 \mid 7}{8 \mid 10}$, for instance, representing a specimen with 9 left and 7 right dorsal plates, and 8 left and 10 right anal plates. In the Plate *dp*. 1 indicates the position of the anterior dorsal and *ap*. 1 of the anterior anal plate.

The most striking peculiarity of the 250 specimens taken as a whole is the very small proportion in which the plates are perfectly symmetrical in arrangement, the number of plates on the left side equalling those on the right in both the dorsal and anal series; this arrangement may be expressed by the diagram $\frac{n \mid n}{m \mid m}$ where n is either less than, equal to, or more than m, e.g. $\frac{8 \mid 8}{9 \mid 9}$, $\frac{9 \mid 9}{9 \mid 9}$,

* See Boulenger: *Ann. Mag. Nat. Hist.*, Series 7, x. p. 303, 1902, where the circumstances which led Mr Holt to make this suggestion are explained. For the material used I am indebted to him and to the Marine Biological Association.

† Boulenger: *loc. cit.* p. 295.

or $\frac{9 \mid 9}{8 \mid 8}$. This symmetry only occurs in 23·6 per cent. of the total number examined.

The arrangement of the plates.

The most convenient way of dealing with the various combinations of numbers of plates found in these specimens seems to be by reference to their symmetry in one or both series.

Among the 250 specimens:—

1. Both series of plates are symmetrical in 59 or 23·6 °/₀.
2. One series is asymmetrical in 112 or 44·8 °/₀.
3. Both series are asymmetrical in 79 or 31·6 °/₀.

1. Among the symmetrical specimens the following arrangements occur:—

$$\left[\frac{n \mid n}{m \mid m}\right].$$

$\frac{8 \mid 8}{7 \mid 7}$	$\frac{7 \mid 7}{8 \mid 8}$	$\frac{8 \mid 8}{8 \mid 8}$	$\frac{9 \mid 9}{8 \mid 8}$	$\frac{8 \mid 8}{9 \mid 9}$	$\frac{9 \mid 9}{9 \mid 9}$	$\frac{10 \mid 10}{9 \mid 9}$	$\frac{8 \mid 8}{10 \mid 10}$	$\frac{9 \mid 9}{10 \mid 10}$
2	1	11	2	25	14	1	1	2

2. Among the 112 asymmetrical in one series the following arrangements occur:—

(*a*) Of the formula $\frac{n+k \mid n}{m \mid m}$, 33 or 13·2 °/₀.

$\frac{8 \mid 7}{8 \mid 8}$	$\frac{9 \mid 7}{8 \mid 8}$	$\frac{9 \mid 8}{8 \mid 8}$	$\frac{10 \mid 9}{8 \mid 8}$	$\frac{8 \mid 7}{9 \mid 9}$	$\frac{9 \mid 7}{9 \mid 9}$	$\frac{9 \mid 8}{9 \mid 9}$	$\frac{10 \mid 8}{9 \mid 9}$	$\frac{10 \mid 9}{9 \mid 9}$	$\frac{9 \mid 8}{10 \mid 10}$	$\frac{10 \mid 8}{10 \mid 10}$
2	1	4	1	3	1	14	2	1	3	1

(*b*) Of the formula $\frac{n \mid n+k}{m \mid m}$, 32 or 12·8 °/₀.

$\frac{7 \mid 8}{8 \mid 8}$	$\frac{7 \mid 9}{8 \mid 8}$	$\frac{8 \mid 9}{8 \mid 8}$	$\frac{7 \mid 8}{9 \mid 9}$	$\frac{8 \mid 9}{9 \mid 9}$	$\frac{8 \mid 10}{9 \mid 9}$	$\frac{9 \mid 10}{9 \mid 9}$	$\frac{8 \mid 9}{10 \mid 10}$	$\frac{8 \mid 10}{10 \mid 10}$	$\frac{9 \mid 10}{10 \mid 10}$	$\frac{9 \mid 11}{10 \mid 10}$
1	1	1	2	17	2	4	1	1	1	1

(*c*) Of the formula $\frac{n \mid n}{m+k \mid m}$, 26 or 10·4 °/₀.

$\frac{8 \mid 8}{8 \mid 7}$	$\frac{7 \mid 7}{9 \mid 8}$	$\frac{8 \mid 8}{9 \mid 8}$	$\frac{9 \mid 9}{9 \mid 8}$	$\frac{8 \mid 8}{10 \mid 8}$	$\frac{8 \mid 8}{10 \mid 9}$	$\frac{9 \mid 9}{10 \mid 9}$	$\frac{10 \mid 10}{10 \mid 9}$	$\frac{9 \mid 9}{12 \mid 9}$
3	1	4	6	1	4	4	2	1

(*d*) Of the formula $\frac{n \mid n}{m \mid m+k}$, 21 or 8·4 °/₀.

$\frac{7 \mid 7}{7 \mid 8}$	$\frac{8 \mid 8}{7 \mid 9}$	$\frac{9 \mid 9}{7 \mid 9}$	$\frac{8 \mid 8}{8 \mid 9}$	$\frac{9 \mid 9}{8 \mid 9}$	$\frac{8 \mid 8}{8 \mid 10}$	$\frac{9 \mid 9}{8 \mid 10}$	$\frac{8 \mid 8}{9 \mid 10}$	$\frac{9 \mid 9}{9 \mid 10}$	$\frac{9 \mid 9}{10 \mid 11}$
1	1	1	5	6	1	1	1	3	1

3. Among the 79 asymmetrical in both series the following arrangements occur:—

(i) Where both series have more plates on the same side, 40 or 16 °/₀.

(*a*) Of the formula $\frac{n+k \mid n}{m+l \mid m}$, 18 or 7·2 °/₀.

$\frac{8 \mid 7}{9 \mid 7}$	$\frac{8 \mid 7}{9 \mid 8}$	$\frac{9 \mid 8}{9 \mid 8}$	$\frac{10 \mid 8}{9 \mid 8}$	$\frac{10 \mid 9}{9 \mid 8}$	$\frac{10 \mid 9}{10 \mid 7}$	$\frac{9 \mid 7}{10 \mid 8}$	$\frac{9 \mid 8}{10 \mid 8}$	$\frac{9 \mid 8}{10 \mid 9}$	$\frac{10 \mid 9}{10 \mid 9}$
1	2	2	1	1	1	1	1	7	1

(*b*) Of the formula $\frac{n \mid n+k}{m \mid m+l}$, 22 or 8·8 °/₀.

$\frac{7 \mid 8}{7 \mid 9}$	$\frac{8 \mid 9}{7 \mid 9}$	$\frac{7 \mid 8}{8 \mid 9}$	$\frac{8 \mid 9}{8 \mid 9}$	$\frac{8 \mid 9}{8 \mid 10}$	$\frac{8 \mid 9}{9 \mid 10}$	$\frac{9 \mid 10}{9 \mid 10}$	$\frac{10 \mid 11}{9 \mid 10}$	$\frac{8 \mid 9}{9 \mid 11}$
1	1	5	6	1	4	2	1	1

(ii) Where the plates of one side preponderate in one series and of the other in the other.

(*a*) Where the total number of plates on the right side equals the total number on the left and the specimen attains a "secondary symmetry," 32 or 12·8 °/₀.

(1) Of the formula $\frac{n+k \mid n}{m \mid m+k}$, 13 or 5·2 °/₀.

$\frac{8 \mid 7}{8 \mid 9}$	$\frac{9 \mid 8}{8 \mid 9}$	$\frac{10 \mid 9}{8 \mid 9}$	$\frac{9 \mid 7}{8 \mid 10}$	$\frac{9 \mid 8}{9 \mid 10}$	$\frac{10 \mid 9}{9 \mid 10}$	$\frac{9 \mid 7}{9 \mid 11}$
1	5	1	1	3	1	1

(2) Of the formula $\frac{n \mid n+k}{m+k \mid m}$, 19 or 7·6 °/₀.

$\frac{7 \mid 8}{9 \mid 8}$	$\frac{8 \mid 9}{9 \mid 8}$	$\frac{7 \mid 8}{10 \mid 9}$	$\frac{8 \mid 9}{10 \mid 9}$	$\frac{9 \mid 10}{10 \mid 9}$	$\frac{8 \mid 10}{11 \mid 9}$
1	13	1	2	1	1

(*b*) Where the plates of one side or the other preponderate there are 7 or 2·8 °/₀ of the following formulæ and arrangements:—

$\frac{n+k}{m}\,\vert\,\frac{n}{m+k+l}$		$\frac{n+k+l}{m}\,\vert\,\frac{n}{m+k}$	$\frac{n}{m+k}\,\vert\,\frac{n+k+l}{m}$		
$\frac{9\,\vert\,8}{9\,\vert\,10}$	$\frac{8\,\vert\,7}{8\,\vert\,10}$	$\frac{9\,\vert\,7}{8\,\vert\,9}$	$\frac{7\,\vert\,9}{9\,\vert\,8}$	$\frac{8\,\vert\,10}{9\,\vert\,8}$	$\frac{8\,\vert\,10}{10\,\vert\,11}$
1	1	2	1	1	1

The following are the particular combinations which occur in more than 5 per cent. of the 250 specimens:—

$\frac{8\,\vert\,8}{9\,\vert\,9}$ in 25 specimens or 10 per cent of the total.

$\frac{8\,\vert\,9}{9\,\vert\,9}$ in 17 „ „ 6·8 „ „

$\frac{9\,\vert\,8}{9\,\vert\,9}$ in 14 „ „ 5·6 „ „

$\frac{9\,\vert\,9}{9\,\vert\,9}$ in 14 „ „ 5·6 „ „

$\frac{8\,\vert\,9}{9\,\vert\,8}$ in 13 „ „ 5·2 „ „

The number of the plates.

Dr Günther in the first edition of the British Museum Catalogue gives the total number of plates in *Z. faber* as 9·8, or 7 occasionally 10 in each dorsal series and 9 in each anal. In the specimens under consideration the maxima and minima observed were:—

Dorsal L. Max. 10 Min. 7 R. Max. 11 Min. 7
Anal „ „ 12 „ 7 „ „ 11 „ 7

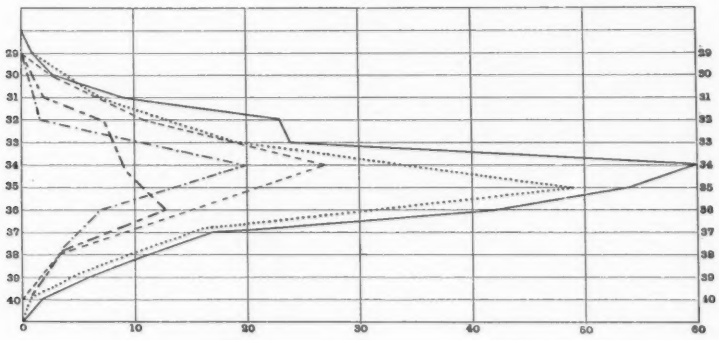
and the variation may be diagrammatically expressed $\frac{10-7\,\vert\,7-11}{12-7\,\vert\,7-11}$.

The average number of plates was 34·6, $\frac{8{\cdot}432\,\vert\,8{\cdot}464}{8{\cdot}864\,\vert\,8{\cdot}840}$, the largest in any individual 40, in two instances, $\frac{10\,\vert\,11}{9\,\vert\,10}$ and $\frac{9\,\vert\,11}{10\,\vert\,10}$, and the smallest 29, $\frac{7\,\vert\,7}{7\,\vert\,8}$.

The accompanying table and diagram show the number of plates occurring in the individuals of each formula, and the nearly regular grouping of them in each instance round a maximum of 34 or 35; (it must be borne in mind that in a specimen primarily or secondarily symmetrical there must be an even number of plates and that in a specimen asymmetrical in one series an even number is very uncommon).

Number of Plates

	29	30	31	32	33	34	35	36	37	38	39	40	Total	Per cent.
1. $\frac{n \mid n}{m \mid m}$ ...	—	3	—	11	—	27	—	15	—	3	—	—	59	23·6
2. (*a*) $\frac{n+k \mid n}{m \mid m}$	—	—	2	1	7	1	15	2	4	1	—	—	33	13·2
(*b*) $\frac{n \mid n+k}{m \mid m}$	—	—	1	1	3	—	17	2	5	1	1	1	32	12·8
(*c*) $\frac{n \mid n}{m+k \mid m}$	—	—	4	—	4	1	10	—	4	—	3	—	26	10·4
(*d*) $\frac{n \mid n}{m \mid m+k}$	1	—	—	1	5	2	7	1	3	—	1	—	21	8·4
3. (i) (*a*) $\frac{n+k \mid n}{m+l \mid m}$	—	—	1	2	—	3	2	9	—	1	—	—	18	7·2
(*b*) $\frac{n \mid n+k}{m \mid m+l}$	—	—	1	5	1	6	1	4	1	2	—	1	22	8·8
(ii) (*a*) (1) $\frac{n+k \mid n}{m \mid m+k}$	—	—	—	1	—	6	—	5	—	1	—	—	13	5·2
(2) $\frac{n \mid n+k}{m+k \mid m}$	—	—	—	1	—	14	—	2	—	2	—	—	19	7·6
(*b*)	—	—	—	—	4	—	2	—	—	—	1	—	7	2·8
Total	1	3	9	23	24	60	54	40	17	11	6	2		
Per cent. ...			3·6	9·2	9·6	24	21·6	16	6·8	4·4	2·4			



Total ——

1. $\frac{n \mid n}{m \mid m}$ - - - - -
2. Asymmetrical in one series* .....
3. (i) Asymmetrical in both† — · — · — ·
3. (ii) Secondarily symmetrical – · – · – · –

* Very few specimens have an even number of plates and these are omitted in tracing the diagram.
† Specimens with an odd number of plates omitted.

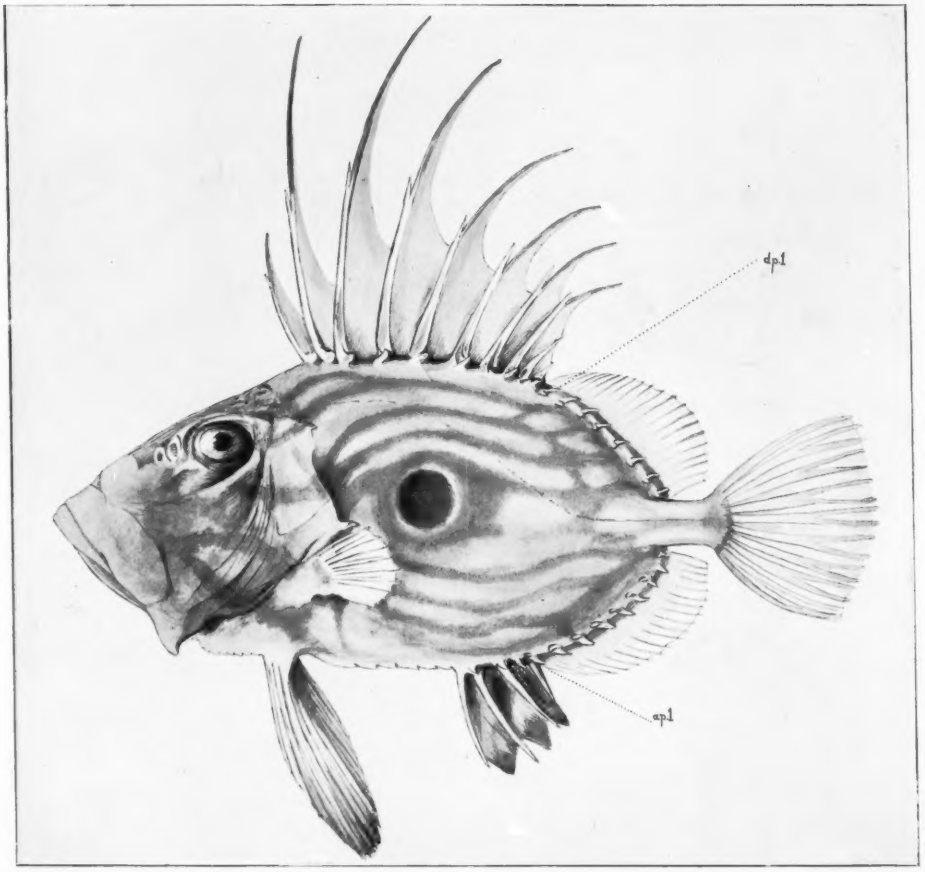
So far as can be judged from such a small number of specimens the following points seem worth noting:—

1. The plates are arranged symmetrically in only 23·6 per cent. of the whole.

2. The average number of plates is 34·6, and the actual numbers of most frequent occurrence 34, 35, and 36 (one of these three occurring in over 60 per cent. of the total).

3. The most frequent combination is $\frac{8 \mid 8}{9 \mid 9}$ which occurs in 10 per cent. of the total; the other most usual combinations are those in which one or both of the upper series are increased to 9.

4. The largest number of plates noted was 40, $\frac{10 \mid 11}{9 \mid 10}$ and $\frac{9 \mid 11}{10 \mid 10}$, and the smallest 29, $\frac{7 \mid 7}{7 \mid 8}$.



Young John Dory, showing position of anterior dorsal and anal plates.